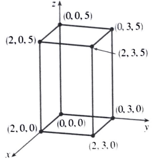
4. The projection of $(2, 3, 5)$ onto the xy-plane is $(2, 3, 0)$; onto the yz-plane, $(0, 3, 5)$; onto the xz-plane, $(2, 0, 5)$.



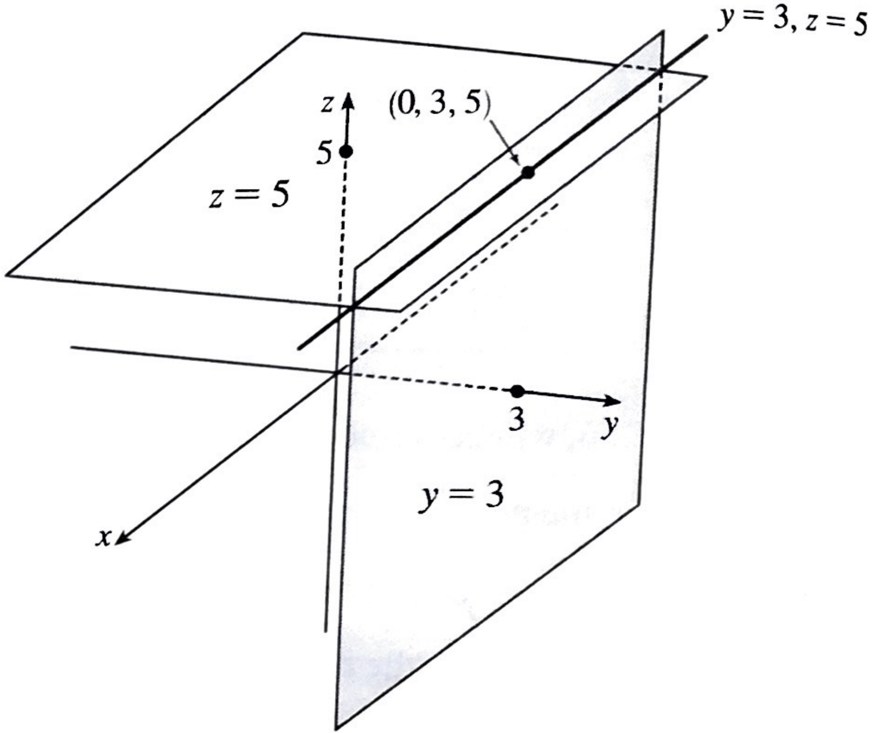
The length of the diagonal of the box is the distance between the origin and $(2, 3, 5)$, given by

$$\sqrt{(2-0)^2 + (3-0)^2 + (5-0)^2} = \sqrt{38} \approx 6.16$$

6. In $\mathbb{R}^3$, the equation $y = 3$ represents a vertical plane that is parallel to the xz-plane and 3 units to the right of it. The equation $z = 5$ represents a horizontal plane parallel to the xy-plane and 5 units above it. The pair of equations $y = 3$, $z = 5$ represents the set of points that are simultaneously on both planes, or in other words, the line of intersection of the planes $y = 3$, $z = 5$.

[continued]

This line can also be described as the set $\{(x, 3, 5) \mid x \in \mathbb{R}\}$, which is the set of all points in $\mathbb{R}^3$ whose x-coordinate may vary but whose y- and z-coordinates are fixed at 3 and 5, respectively. Thus the line is parallel to the x-axis and intersects the yz-plane in the point $(0, 3, 5)$.



20. Completing squares in the equation $3x^2 + 3y^2 - 6y + 3z^2 - 12z = 10$ gives

$3x^2 + 3(y^2 - 2y + 1) + 3(z^2 - 4z + 4) = 10 + 3 + 12 \Rightarrow 3x^2 + 3(y-1)^2 + 3(z-2)^2 = 25 \Rightarrow$

$x^2 + (y-1)^2 + (z-2)^2 = \frac{25}{3}$, which we recognize as an equation of a sphere with center $(0, 1, 2)$ and radius

$\sqrt{\frac{25}{3}} = 5/\sqrt{3}$.

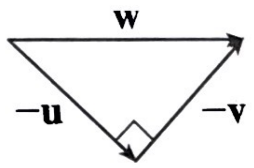
4. (a) The initial point of $\overrightarrow{BC}$ is positioned at the terminal point of $\overrightarrow{AB}$, so by the Triangle Law the sum $\overrightarrow{AB} + \overrightarrow{BC}$ is the vector with initial point A and terminal point C, namely $\overrightarrow{AC}$.

(b) By the Triangle Law, $\overrightarrow{CD} + \overrightarrow{DB}$ is the vector with initial point C and terminal point B, namely $\overrightarrow{CB}$.

(c) First we consider $\overrightarrow{DB} - \overrightarrow{AB}$ as $\overrightarrow{DB} + \left(-\overrightarrow{AB}\right)$. Then since $-\overrightarrow{AB}$ has the same length as $\overrightarrow{AB}$ but points in the opposite direction, we have $-\overrightarrow{AB} = \overrightarrow{BA}$ and so $\overrightarrow{DB} - \overrightarrow{AB} = \overrightarrow{DB} + \overrightarrow{BA} = \overrightarrow{DA}$.

(d) We use the Triangle Law twice: $\overrightarrow{DC} + \overrightarrow{CA} + \overrightarrow{AB} = \left(\overrightarrow{DC} + \overrightarrow{CA}\right) + \overrightarrow{AB} = \overrightarrow{DA} + \overrightarrow{AB} = \overrightarrow{DB}$.

8. We are given $\mathbf{u} + \mathbf{v} + \mathbf{w} = \mathbf{0}$, so $\mathbf{w} = (-\mathbf{u}) + (-\mathbf{v})$. (See the figure.)



Vectors $-\mathbf{u}$, $-\mathbf{v}$, and $\mathbf{w}$ form a right triangle, so from the Pythagorean Theorem

we have $|-\mathbf{u}|^2 + |-\mathbf{v}|^2 = |\mathbf{w}|^2$. But $|-\mathbf{u}| = |\mathbf{u}| = 1$ and $|-\mathbf{v}| = |\mathbf{v}| = 1$ so $|\mathbf{w}| = \sqrt{|-\mathbf{u}|^2 + |-\mathbf{v}|^2} = \sqrt{2}$.

22. $\mathbf{a} + \mathbf{b} = \langle 8 + 5, 1 + (-2), -4 + 1 \rangle = \langle 13, -1, -3 \rangle$

$4\,\mathbf{a} + 2\,\mathbf{b} = 4\,\langle 8, 1, -4 \rangle + 2\,\langle 5, -2, 1 \rangle = \langle 32, 4, -16 \rangle + \langle 10, -4, 2 \rangle = \langle 42, 0, -14 \rangle$

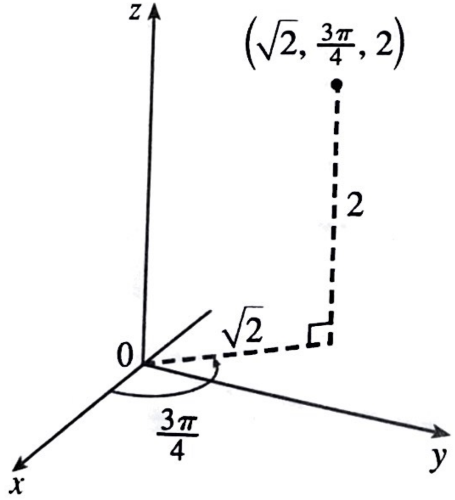
$|\mathbf{a}| = \sqrt{8^2 + 1^2 + (-4)^2} = \sqrt{81} = 9$

$|\mathbf{a} - \mathbf{b}| = |\langle 8 - 5, 1 - (-2), -4 - 1 \rangle| = |\langle 3, 3, -5 \rangle| = \sqrt{3^2 + 3^2 + (-5)^2} = \sqrt{43}$

6. (a) $x = \sqrt{3}$ and $y = -1$ $\Rightarrow$ $r = \sqrt{\left(\sqrt{3}\right)^2 + (-1)^2} = 2$ and $\tan\theta = \dfrac{-1}{\sqrt{3}}$ $[\theta = -\frac{\pi}{6} + n\pi]$. Since $\left(\sqrt{3}, -1\right)$ is in the fourth quadrant, the polar coordinates are (i) $\left(2, \frac{11\pi}{6}\right)$ and (ii) $\left(-2, \frac{5\pi}{6}\right)$.

(b) $x = -6$ and $y = 0$ $\Rightarrow$ $r = \sqrt{(-6)^2 + 0^2} = 6$ and $\tan\theta = \frac{0}{-6} = 0$ $[\theta = n\pi]$. Since $(-6, 0)$ is on the negative x-axis, the polar coordinates are (i) $(6, \pi)$ and (ii) $(-6, 0)$.

2. (a)

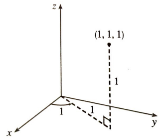


$$x = \sqrt{2}\cos\frac{3\pi}{4} = \sqrt{2}\left(-\tfrac{\sqrt{2}}{2}\right) = -1,$$

$$y = \sqrt{2}\sin\frac{3\pi}{4} = \sqrt{2}\left(\tfrac{\sqrt{2}}{2}\right) = 1, \text{ and } z = 2,$$

so the point is $(-1, 1, 2)$ in rectangular coordinates.

(b)



$x = 1\cos 1 = \cos 1$, $y = 1\sin 1 = \sin 1$, and $z = 1$, so the point is $(\cos 1, \sin 1, 1) \approx (0.54, 0.84, 1)$ in rectangular coordinates.

6. Since $\theta = \frac{\pi}{6}$ but r and z may vary, the surface is a vertical plane including the z-axis and intersecting the xy-plane in the line $y = \frac{1}{\sqrt{3}}x$. (Here we are assuming that r can be negative; if we restrict $r \geq 0$, then we get a half-plane.)