Homework 4

(a) R should be a linear ordering because for any $x, y \in P$ and $m, n \in \mathbb{N}$,

xRy is true or yRx is true | One element is not smaller than the other
mRn is true or nRm is true | for any two elements chosen in the set.

Also, P is a partial ordering. Therefore, P is a linear ordering.

(b) (i) for (a), the sentence follows:

For any x in the set X, there exists an element y that is also in X such that y is not smaller than x.

A sentence about $\mathbb{N}$: For any positive integer x, there exists a positive integer y such that y is not smaller than x.

A sentence about P: For any population x of the United States, there exists a population y of the United such that y is not smaller than x.

for (b), the sentence is

There exists y in the set X such that for any x in the set X, y is not smaller than x.

A sentence about $\mathbb{N}$: There exists a positive integer y that is ~~bigger~~ not smaller than ~~any other element x in N~~ any positive integer x. (*)

A sentence about P: There exists a population of the US y such that it is not smaller than any population of the US x.

(ii) (*) is not possible. Suppose that we can find such an integer y. Then, $y+1$ is still a positive integer and $y+1 > y$, which contradicts with statement that y is not smaller than any positive integer.

(c) The two ~~relation~~ sentences are equivalent

3.9 Let $p(x)$ be a formula in one variable Universe $U = \mathbb{Z}$

p: $x \bmod 2 = 0$,

3.10 $P(x)$: $\exists n \in \mathbb{N}$ $x = n^2$

3.13 a) d) e)

3.14 c) Negation: $(\forall x \in \mathbb{Z})(\exists y \in \mathbb{Z})$ $x > y$ True d) $(\exists y \in \mathbb{Z})(\forall x \in \mathbb{Z})$ $x > y$ False

~~d) $(\exists \varepsilon > 0)(\forall \delta > 0)(\exists x \in \mathbb{R})$ $[0 < |x-1| < \delta] \wedge [|x^2 - 1| \geq \varepsilon]$~~

3.17 Contrapositive Converse

(i) ~~All immortal a~~

Any immortal is not a man ~~There is a man that is immortal~~

All mortals are men

(ii) ~~If there is a thing I don't say, I don't mean it.~~

If I don't say it, I don't mean it. ~~There is something I mean but I don't say it.~~ What I say is what I mean

(iii) If a function doesn't attain its maximum on the interval $[0,1]$, then it's not a continuous function on the interval $[0,1]$

~~There exists a continuous function on the interval $[0,1]$ that doesn't attain its maximum~~

If a function attains its maximum on $[0,1]$ then it's continuous on $[0,1]$.

(iv) If the sum of the angles of a polygon is not 180°, then it's not a triangle.

~~The sum of the angles of some triangle is not 180°~~

If the sum of the angles of a polygon is 180° then it's a triangle.

3.5 II ~~part~~

a) $P \wedge Q$ ~~$\equiv (\neg P) \vee Q$~~

$\equiv \neg(\neg P \vee \neg Q)$

b) $P \vee Q \equiv (\neg(\neg P)) \vee (\neg(\neg Q))$ c) $(P \Rightarrow Q) \equiv (\neg P \vee Q)$