6.3 ≤ is transitive: For sets X, Y, Z

if X ≤ Y and Y ≤ Z, 

There is an injection \( f: X \to Y \) 

There is an injection \( g: Y \to Z \)

Then \( g \circ f: X \to Z \) is an injection. Therefore, \( X \leq Z \).

≤ is transitive.

≤ is reflexive: \( X \leq X \) For any set X

Let \( f \) be the identity function. \( \forall x \in X, f(x) = x \).

Then, \( f: X \to X \) is an injection.

Therefore, \( X \leq X \)

6.5 \(|P(\{1, 2, 3, \ldots, n-1\})| = 2^n - 1\) 

\( P(X) \): power set of \( X = \{Y \mid Y \subseteq X \} \)

\( P(X) \) s.t. \(|P(X)| = 0 \)

\(|P(X)| = 1 \)

\(|P(X)| = 2 \)

\(|P(X)| = n \)

So \(|P(\{1, 2, 3, \ldots, n-1\})| = 2^{n-1} = (1+1)^{n-1} \)

6.13 If \( |Y| \leq |X| \), then there is a surjection \( f: X \to Y \).

Since \( |Y| \leq |X| \), we have \( Y \leq X \). There exists an injection \( g: Y \to X \)

Let \( A = g(Y) \) and let \( y \) be an element in \( Y \)

Define \( f_A = \begin{cases} g^{-1}(x) & \text{if } x \in A \\ y & \text{if } x \notin A \end{cases} \)

\( f \) is a surjection.
6.17  a) \( f(x) = \frac{d-c}{b-a} x + \frac{bc-ad}{b-a} \) \( f: (a,b) \to (c,d) \) 

\( f \) is a bijection, so \((a,b)\) is bijective with \((c,d)\).

b) \( f(x) = \frac{d-c}{b-a} x + \frac{bc-ad}{b-a} \) \( f: [a,b] \to [c,d] \) is a bijection.

d) Since \( c < d \), we can find \( e, f \) s.t. \( c < e < f < d \) 

\( \exists \) an injection from \((a,b) \to (e,f)\)

\( \Rightarrow \exists \) an injection from \((a,b) \to [c,d] \) since \( (e,f) \subseteq [c,d] \)

Since \( a < b \), we can find \( g, h \) s.t. \( a < g < h < b \)

\( \exists \) an bijection from \([c,d] \to [g,h] \)

\( \Rightarrow \exists \) an injection from \([c,d] \to (a,b) \) since \([g,h] \subset (a,b)\)

Therefore, \((a,b)\) is bijective with the closed interval \([c,d] \).

e) \( |[0,1]| = |\mathbb{R}| \) since \([0,1] \subset \mathbb{R} \)

\( \exists \) an injection from \([0,1] \to \mathbb{R} \) \( (x) \)

Let \( f \) be a function from \( \mathbb{R} \) to \([0,1] \)

\[ f(x) = \begin{cases} 
\frac{x}{x+1} & \text{if } x \geq 0 \\
\left\lfloor \frac{|x|}{|x|+1} \right\rfloor & \text{if } x < 0 
\end{cases} \]

Then, \( f \) is an injection from \( \mathbb{R} \) to \([0,1] \)

Since there is a bijection from \([-1,1] \to [0,1] \) by \( b \), we know

\( \exists \) a bijection from \( \mathbb{R} \) to \([0,1] \) \( (x) \)

by \((x)\) and \((x)\), we know \( \exists \) a bijection from \([0,1] \) to \( \mathbb{R} \)

So, \( |[0,1]| = |\mathbb{R}| \)
3. Construct an explicit bijection between $(0,1)$ and $[0,1]$

$$f(x) = \begin{cases} 
0 & \text{if } x = \frac{1}{2} \\
1 & \text{if } x = \frac{1}{3} \\
\frac{n-2}{n} & \text{if } x = \frac{1}{n} \\
x & \text{otherwise}
\end{cases} \quad f : (0,1) \to [0,1]$$

4. (a) The set of finite subsets of $\mathbb{N}$ has the same cardinality as

(b) infinite subsets of $\mathbb{N}$ $P(\mathbb{N})$

(c) finite sequence of elements of $\mathbb{N}$