Homework, WEEK 1

1.1 $\{n \in \mathbb{N} \mid n \text{ is odd and } n = k(k+1) \text{ for some } k \in \mathbb{N}\}$

Proof: 1) If k is odd, then $k = 2m+1$ for some $m \in \mathbb{N}$.

Therefore, $n = k(k+1) = (2m+1)(2m+1+1) = (2m+1)(2m+2)$

$n = 2[(m+1)(2m+1)]$ is even.

There doesn't exist any odd n.

2) If k is even, then $k = 2m$ for some $m \in \mathbb{N}$

$n = k(k+1) = 2m(2m+1) = 2[m(2m+1)]$ is even and can't be odd.

Since $k \in \mathbb{N}$, k is even or odd, and thus $n = k(k+1)$ is even in both cases. Therefore, $\{n \in \mathbb{N} \mid n \text{ is odd and } n = k(k+1) \text{ for some } k \in \mathbb{N}\}$

$= \emptyset$

1.2 (i) $(X \cup Y)^c = X^c \cap Y^c$

Proof: If $p \in (X \cup Y)^c$, then $p \notin (X \cup Y)$.

$\Rightarrow p \notin X$ and $p \notin Y \Rightarrow p \in X^c$ and $p \in Y^c \Rightarrow p \in X^c \cap Y^c$

Therefore, $(X \cup Y)^c \subset X^c \cap Y^c$

If $p \in X^c \cap Y^c$, then $p \in X^c$ and $p \in Y^c \Rightarrow p \notin X$ and $p \notin Y$

$\Rightarrow p \notin (X \cup Y) \Rightarrow p \in (X \cup Y)^c$. Thus, $X^c \cap Y^c \subset (X \cup Y)^c$

Since $(X \cup Y)^c \subset X^c \cap Y^c$ and $X^c \cap Y^c \subset (X \cup Y)^c$, we have

$(X \cup Y)^c = X^c \cap Y^c$

1.4 (ii) $X \times Y \times Z$

$X = \lceil 2 \rceil = \{0, 1\}$

$Y = \lceil 3 \rceil = \{0, 1, 2\}$

$Z = \lceil 1 \rceil = \{0\}$

$X \times Y \times Z$

$= \{(0,0,0), (0,1,0), (0,2,0), (1,0,0), (1,1,0), (1,2,0)\}$

(iii) $X \times Y \times Z \times \emptyset = \emptyset$ (v) $X^n = \{(a_1, a_2, \ldots, a_n) \mid a_i \in \{0,1\}\}$

1.7 Describe all possible intervals in $\mathbb{Z}$. Check P. 38 of the textbook

$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\ldots\}$

All possible intervals: ~~$[k,k]$, where $k \in \mathbb{Z}$~~

a subset of $\mathbb{Z}$ in the form $[a, a+1, \ldots, b-1, b]$ where a and b are integers.

1.10 (i) addition, multiplication and (exponentiation) are (depending on whether 0^0 is defined)

(ii) addition, subtraction, multiplication are

(v) addition, multiplication, division and exponentiation are

15 f is not a function on X ~~or~~ f is a function on Y and on Z

g is ~~or~~ a function on X and Y and Z

the complement of Z in X is all isosceles triangles.

$Y \cap Z^c$ is all isosceles right-angled triangles

1.16 (iii) $\bigcup_{0<t\leq 10} X_t = (-10, 10)$ $\bigcup_{0<t\leq 10} Y_t = [-10, 10]$

(iv) $\bigcap_{t\geq 10} X_t = (-10, 10)$, $\bigcap_{t\geq 10} Y_t = [-10, 10]$

1.19 Injective: ~~(i)~~ (ii)

Surjective: (i) (ii)