Title: Arithmetic progressions and nilmanifolds

Abstract: The connection between ergodic theory and combinatorial number theory was first introduced by Furstenberg in his seminal paper in 1977. Furstenberg gave an ergodic theoretic proof of a famous theorem of Szemeredi on the existence of arbitrarily long arithmetic progressions in sets of positive upper density in the integers. Furstenberg's proof introduced 'multiple' ergodic averages, which deal with expressions involving the states of a dynamical system along a 'nice' sequence of times. It turns out that the limiting behavior of these averages is of algebraic nature - more precisely, it involves the dynamics of translations on homogeneous spaces of nilpotent Lie groups.

   We will describe some of the ideas in the study of multiple averages and point out analogies with work of Green and Tao related to quadratic Fourier analysis.