Speaker: Charles Weibel

Title: The Norm Residue Theorem in Motivic Cohomology

Abstract: The Norm Residue Theorem says that the etale cohomology of a field has a presentation with units as generators and simple quadratic relations. The ring with this presentation was described by Milnor in 1970 and this theorem had been conjectured by Milnor (at the prime 2) and Bloch-Kato (at odd primes). It implies several other striking results, involving motivic cohomology, algebraic K-theory and the Riemann zeta function of number fields. The proof of the theorem spans about 20 research papers, with the main steps due to Voevodsky and Rost.

The first part of this talk will be a non-technical overview of the ingredients that go into the proof, and what the consequences are.

Here is a fun consequence of all this. We now know the first 20,000 groups $K_n(\mathbb{Z})$ of the integers, except when 4 divides n. The assertion that these groups are zero when 4 divides n ($n > 0$) is equivalent to Vandiver's Conjecture (in number theory), and if it holds then we have fixed Kummer's 1849 "proof" of Fermat's Last Theorem. If any of them are nonzero, then the smallest prime dividing the order of this group is at least 16,000,000.