

# Special cycles on Shimura varieties

Wei Zhang

On a Shimura variety  $X$  attached to a reductive group  $G$ , one may look at the Hecke action on the space  $CH^*(X)$  of algebraic cycle classes and ask the following question: is this action “modular” in the sense that any irreducible Hecke submodule of  $CH^*(X)$  is isomorphic to a Hecke module coming from an automorphic representation of  $G$ ?

In this talk we will look at a subspace of  $CH^*(X)$  of “special cycles” that come from sub-Shimura varieties and their Hecke translations. The typical examples are Heegner points on a modular curve.

For Shimura varieties of orthogonal type, we can prove that the space of special cycles is finite dimensional and we answer affirmatively a conjecture of Kudla that the generating functions of these special cycles are Siegel modular forms. From this, we can then answer the question we raised above using “theta duality” between orthogonal and symplectic groups.