Speaker: Angelo Vistoli

Title: Fundamental gerbes

Abstract: Let X be a connected and geometrically reduced variety over a field k, with a fixed rational point x_0 in $X(k)$. Nori defined a profinite group scheme $N(X, x_0)$, usually called Nori's fundamental group scheme, with the property that homomorphisms $N(X, x_0)$ to a fixed finite group scheme G correspond to G-torsors $P \to X$, with a fixed rational point in the inverse image of x_0 in P. If k is algebraically closed of characteristic 0 this coincides with Grothendieck's fundamental group, but is in general very different. Nori's main theorem is that if X is complete, the category of finite-dimensional representations of $N(X, x_0)$ is equivalent to an abelian subcategory of the category of vector bundles on X, the category of essentially finite bundles.

During the pre-talk, I will recall the basics of the theory of group schemes and torsors.

In the first seminar I will give a detailed description of Nori's results. Then I will explain my work in collaboration with Niels Borne, from the University of Lille, in which we extend them by removing the dependence on the base point, substituting Nori's fundamental group with a gerbe (in characteristic 0 this had already been done by Deligne), and give a simpler definition of essentially finite bundle. I will dedicate a substantial part of the seminar to explain what a gerbe is, and explain why the language of gerbes is well suited to remove the dependence of the group on the base point.

In the second seminar I will try to give an idea of the proof of the main theorem, and explain how our results lead to a very natural formulation of Grothendieck's famous section conjecture.