Speaker: Andrew Sutherland

Title: Strong arithmetic equivalence

Abstract: Number fields with same Dedekind zeta function are said to be arithmetically equivalent. Such number fields necessarily have the same degree, discriminant, signature, and Galois closure, but need not be isomorphic, and may have distinct class numbers, regulators, and rings of adeles. Motivated by a recent result of Prasad, I will discuss two stronger notions of arithmetic equivalence, both of which force isomorphisms of all the invariants listed above. Up to now the only known construction of non-isomorphic number fields satisfying either of these stronger notion of equivalence relies on a particular group-theoretic construction due to Leonard Scott that can be used to prove the existence of pairs of non-isomorphic number fields of degree 203 satisfying both notions of strong arithmetic equivalence.

I will present work in progress that includes the construction of several infinite families of such fields (one of which includes Scott's example), explicit pairs of non-isomorphic number fields of degree 32 that satisfy one, but not both, notions of strong arithmetic equivalence (thereby proving that the two notions are not equivalent), and explicit examples of non-isomorphic number fields of degree 96 that satisfy both. These results can also be used to construct pairs of curves with isomorphic Jacobians (via Prasad), pairs of isospectral Riemannian manifolds (via Sunada), and pairs of isospectral graphs (via Halbeisen-Hungerbuhler), however in these applications, unlike the number field case, additional constraints are required to ensure non-isomorphism.