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Title: Optimal strong approximation for quadratic forms

Abstract: For a non-degenerate integral quadratic form $F(x_1, \ldots, x_d)$ in 5 (or more) variables, we prove an optimal strong approximation theorem. Fix any compact subspace $\Omega \subset \mathbb{R}^d$ of the affine quadric $F(x_1, \ldots, x_d) = 1$. Suppose that we are given a small ball B of radius $0 < r < 1$ inside Ω, and an integer m. Further assume that N is a given integer which satisfies $N \gg (r^{-1}m)^{4+\epsilon}$ for any $\epsilon > 0$. Finally assume that we are given an integral vector $(\lambda_1, \ldots, \lambda_d)$ mod m. Then we show that there exists an integral solution $x = (x_1, \ldots, x_d)$ of $F(x) = N$ such that $x_i \equiv \lambda_i$ mod m and $\frac{x}{\sqrt{N}} \in B$, provided all the local conditions are satisfied. We also show that 4 is the best possible exponent. Moreover, for a non-degenerate integral quadratic form $F(x_1, \ldots, x_4)$ in 4 variables we prove the same result if $N \geq (r^{-1}m)^{6+\epsilon}$ and some non-singular local conditions for N are satisfied. Based on some numerical experiments on the diameter of LPS Ramanujan graphs, we conjecture that the optimal strong approximation theorem holds for any quadratic form $F(X)$ in 4 variables with the optimal exponent 4.