Speaker: Gopal Prasad

Title: Arithmetic groups with minimal cohomology and analogues of fake projective planes

Abstract: The cohomology ring of an arithmetic subgroup Γ of a connected semi-simple Lie group G contains the cohomology ring of the compact dual X_u of the symmetric space X of G. We will say that Gamma has minimal cohomology if its cohomology coincides with that of X_u . In a recent joint work with Sai-Kee Yeung we have determined all G, with X hermitian, which can contain a torsion-free arithmetic subgroup with minimal cohomology. Our earlier results in this direction led to classification of fake projective planes which are smooth projective complex algebraic surfaces (different from \mathbb{P}^2) with same Betti numbers as the complex projective plane \mathbb{P}^2 . We also found the first examples of fake \mathbb{P}^4 and fake Grassmanians $\operatorname{Gr}_{2,5}$. The proofs involve number theoretic estimates of discriminants, class numbers, regulators, and explicit values of zeta and L-functions. We use a formula for the volume of $X \wedge \Gamma$ and the Bruhat-Tits theory of reductive p-adic groups crucially.