Speaker: Hector Pasten

Title: Shimura curves and the *abc* conjecture

Abstract: I'll explain some recent unconditional progress on the *abc* conjecture. Elliptic curves over the rationals admit maps from various Shimura curves, and the comparison ratio of the degree of these maps recovers important information on *abc*-triples. On the other hand, this ratio can be controlled by the Arakelov height of CM points. This requires a number of tools: zero-density estimates for L-functions, integral models for various objects, Galois representations, and some complex-analytic estimates. The final outcome is an unconditional estimate for the product of p-adic valuations of *abc*-triples, which lies beyond the reach of existing methods in the context of the *abc* conjecture such as linear forms in logarithms. Our methods also yield other results. For instance, for totally real fields F of bounded degree, we prove that the Faltings height of modular elliptic curves E over F is bounded linearly on log(modular degree of E) + log(Disc. of F). The logarithmic dependence of the discriminant of F can be seen as evidence towards Vojta's conjecture on algebraic points of bounded degree.