SPEAKER: Paula Tretkoff

TITLE: Transcendence properties of hypergeometric functions and monodromy

ABSTRACT: The exceptional set of a function is the set of algebraic points at which the function assumes algebraic values. Many results in transcendental number theory amount to determining the exceptional set of a classical transcendental function. For example, Lindemann proved in 1882 that the exceptional set of exp(z) consists only of z=0, which implies that both e and pi are transcendental numbers. In 1929, C-L Siegel suggested studying the exceptional set of so-called "G-functions" of which the classical Gauss hypergeometric function is an example. In this lecture, we report on joint work with M.D. Tretkoff on the determination of the exceptional set of the Appell-Lauricella hypergeometric functions of n complex variables, in the case where the monodromy group is a lattice in PU(1,n). We show that the exceptional set is Zariski dense in its space of regular points if and only if the monodromy group is arithmetic. The case n=1 corresponds to the Gauss hypergeometric functions with monodromy group a Fuchsian triangle group, where the results are due to Wolfart, Cohen (me!) and Wustholz. These results are related to the Andre–Oort Conjecture on the Zariski-density of complex multiplication points on subvarieties of Shimura varieties. The case where the monodromy group is not a lattice, and n¿1, is still open and is related to conjectures of Pink generalizing that of Andre–Oort. Time permitting, we will also mention some related results, joint with Shiga and Wolfart, on modular functions.