Speaker: Marcin Mazur

Title: Minimal number of generators and density of generating sets of algebras finite over the integers

Abstract: Consider the following questions:

1. What is the smallest number of elements needed to generate the product of 769 copies of the ring of integral 3×3 matrices considered as an algebra over the rational integers? (Answer: 3; it is 2 for the product of 768 copies though).

2. What is the probability that 2 random 3×3 matrices generate the ring of integral 3×3 matrices (answer: $\frac{1}{\zeta(2)^2\zeta(3)}$, where $\zeta(s)$ is the Riemann zeta function).

I will discuss how to answer such questions for algebras finite over the integers. The talk will be based on work I did on this topic jointly with B. Petrenko and R. Kravchenko. In the preliminary part I may discuss some classical results in group theory which motivated some of our questions.