

Title: INVARIANT MEASURES AND THE SET OF EXCEPTIONS TO THE
LITTLEWOOD CONJECTURE

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Abstract:

There is a long and rich tradition of applying dynamical methods to number theory. In many of these applications, a key role is played by the space

$$X = SL(k, R)/SL(k, Z)$$

of unimodal lattices in R^k .

A basic open problem is the classification of measures on X invariant under the action of the group A of $k \times k$ positive diagonal matrices. Conjectures of Margulis and Katok-Spatzier imply that all such measures are algebraic, and proving this conjecture would have striking number theoretical applications, in particular would resolve the long-standing Littlewood's conjecture regarding simultaneous Diophantine approximations.

Recently, there has been substantial progress towards this conjecture by several authors, which in particular established special cases of the quantum unique ergodicity conjecture. Building upon these works, in joint work with Einsiedler and Katok we prove that any A invariant measure on X for which there is an element of A acting with positive entropy is algebraic. This implies that the set of exceptions to Littlewood's conjecture has Hausdorff dimensions zero.