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Title: An Algebraic Proof of Thurston's Rigidity

Abstract: Thurston's rigidity is a foundational result in complex dynamics, regarding so-called postcritically finite (PCF) algebraic morphisms from $\mathbb{P}^1$ to itself; those are maps ϕ such that, for every critical point ζ of ϕ, we have $\phi^m(\zeta) = \phi^n(\zeta)$ for some distinct iterates ϕ^m, ϕ^n. Thurston's rigidity states that, over the complex numbers, if we fix the degree of ϕ and the integers m and n for every critical point, then there are finitely many PCF maps, with one well-understood exception, consisting of maps coming from multiplication on elliptic curves. While the result is entirely algebraic, the proof is analytic, and fails in positive characteristic. We give a proof of rigidity valid in characteristic larger than the degree of ϕ, for a large subset of moduli space (a countable union of codimension-one subsets). We also discuss the question of infinitesimal rigidity: namely, whether the locus of PCF maps in moduli space is reduced. Folklore in complex dynamics is that the answer is yes, but in positive characteristic, there are some exceptions, modulo what appears to be sparse but arbitrarily large primes.