ABSTRACT:

The coding theory problem is to produce "large" subspaces of a vector space over a finite field without "short" vectors (vectors with few nonzero coordinates). One standard method for doing this, due to Goppa, involves using regular functions on algebraic varieties. This is usually done with curves, but recently Voloch and Zarzar have noticed some advantages in using surfaces instead, particularly surfaces of low Picard number. This leads to the question of how to determine, or at least bound from above, the Picard number of a given "random" surface over a finite field. We describe a strategy for doing this in the case of a smooth surface in projective 3-space, by computing p-adic cohomology on the affine complement using a method due to Griffiths; a recent implementation (in conjunction with Tim Abbott and David Roe) has succeeded in a few examples.