Speaker: Peter Humphries

Title: Quantum unique ergodicity in almost every shrinking ball

Abstract: A well-known conjecture of Berry states that eigenfunctions f of the Laplacian on a finite volume negatively curved manifold M should behave like random waves as the eigenvalue λ_f tends to infinity. One manifestation of this conjecture is quantum unique ergodicity on configuration space, which states that the probability measures $|f|^2 \, d\mu$ converge weakly to the uniform measure $d\mu$ on M. For $M = \Gamma\backslash\mathbb{H}$, these eigenfunctions are Maass forms, and this conjecture is a celebrated theorem of Lindenstrauss and Soundararajan.

It is natural to ask whether equidistribution of the measures $|f|^2 \, d\mu$ still occurs in balls centred at fixed points in $\Gamma\backslash\mathbb{H}$ whose radii shrink as λ_f tends to infinity. We show that if the radius shrinks faster than the Planck scale $1/\sqrt{\lambda_f}$, equidistribution may fail, and we discuss how to prove (conditional or unconditional) results towards equidistribution for balls shrinking at any scale larger than the Planck scale that are centred at almost every point in $\Gamma\backslash\mathbb{H}$.