

Speaker: Ehud Hrushovski

Title: Towards a model theory of global fields

Abstract: The logic of diophantine or geometric structures appears to exhibit a sharp local-global dichotomy. For local or generic geometry, there are many *quantifier-elimination* results, describing an arbitrary definable set in geometric terms, and thus opening the way to a model-theoretic analysis. This includes Tarski's theorem in real semi-algebraic geometry, and the Ax–Kochen theorem on the asymptotic theory of the p -adics. For global fields, all known results go in the opposite direction of undecidability; the first such statement is Gödel's interpretation of syntax (and finite mathematics in general) in the ring of integers. However the reason for the undecidability appears to reside in the accidental nature of finite sets, rather than with the adelic geometry that is generally used to analyze number-theoretic problems. One could conjecture that the parts of adelic geometry that are functorial with respect to finite field extensions should admit a model-theoretic analysis. The talk will report on a research program, joint with Itai Ben-Yaacov, with this aim. Existing results are mostly restricted to the function field version, and include the existential closedness of $k(t)^{\text{alg}}$ as an adelic field in an appropriate language.