

Speaker: Robert Hough

Title: Covering systems of congruences

Abstract: A distinct covering system of congruences is a list of congruences

$$a_i \pmod{m_i}, \quad i = 1, 2, \dots, k$$

whose union is the integers. Erdős asked if the least modulus m_1 of a distinct covering system of congruences can be arbitrarily large (the minimum modulus problem for covering systems) and if there exist distinct covering systems of congruences all of whose moduli are odd (the odd problem for covering systems). I'll discuss my proof of a negative answer to the minimum modulus problem, and a quantitative refinement with Pace Nielsen that proves that any distinct covering system of congruences has a modulus divisible by either 2 or 3. The proofs use the probabilistic method and in particular use a sequence of pseudorandom probability measures adapted to the covering process. Time permitting, I may briefly discuss a reformulation of our method due to Balister, Bollobás, Morris, Sahasrabudhe and Tiba which solves a conjecture of Shinzel (any distinct covering system of congruences has one modulus that divides another) and gives a negative answer to the square-free version of the odd problem.