SCHEMES OF LATTICES

WILLIAM J. HABOUSH

Let k denote the algebraic closure of the field with p elements. Then K, the fraction field of the ring of Witt vectors over k is the local Hilbert class field. Let $\mathcal{O}$ denote ring of Witt vectors. Fix a free $\mathcal{O}$ submodule of maximal rank in K^n and call it F. A lattice is a free $\mathcal{O}$-submodule of K^n of maximal rank and it is special if its n'th exterior power is equal to the top exterior power of F in the n'th exterior power of K^n. I will show how the set of special lattices K^n which are contained in $p^{-r}F$ can be represented as a finite projective variety denoted $\mathbb{L}_n^r(K)$. There are natural inclusions among these schemes and I will show that the Picard group of each is $\mathbb{Z}$ and that the canonical generator of $\mathrm{Pic}(\mathbb{L}_n^{r+1})$ restricts to the canonical generator of $\mathrm{Pic}(\mathbb{L}_n^r)$ so that there is a consistent system of projective embeddings. I will describe the geometry of these schemes and special properties such as normality and I will try to describe potential applications of these results to algebraic number theory. I will also describe natural generalizations of the constructions involved.

E-mail address: `haboush@math.uiuc.edu`