Speaker: Philipp Habegger

Title: Small Height and Infinite Non-abelian Extensions

Abstract: The absolute, logarithmic Weil height is non-negative and vanishes precisely at 0 and at the roots of unity. Moreover, when restricted to a number field there are no arbitrarily small, positive heights. Amoroso, Bombieri, David, Dvornicich, Schinzel, Zannier and others exhibited many infinite extensions of the rationals with a height gap. For example, the maximal abelian extension of any number field has this property. To see a non-abelian example, let E be an elliptic curve defined over the rationals without complex multiplication. The field K generated by all complex points of E with finite order is an infinite extension of the rationals. Its Galois group contains no commutative subgroup of finite index. In the talk, I will sketch a proof that K contains no elements of sufficiently small, positive height.

A similar phenomenon was studied by M. Baker, Gubler, Silverman, Zhang and others on replacing the Weil height by the Neron-Tate height on an abelian variety. In situation at hand, $E(K)$ contains no points of sufficiently small, positive Neron-Tate height. If time permits, I will discuss the implications of this result for the group structure of $E(K)$.