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Moment integrals for $GL(m) \times GL(n)$: toward convexity breaking

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(This is part of a project with A. Diaconu and D. Goldfeld.)

A family of essentially elementary kernels (construable as Poincaré series) is described, which, integrated against $|f|^2$ for a cuspform f on $GL(n)$, yield averages of weighted critical-line integral moments of L-functions $L(*, f \otimes F)$ for cuspforms F on $GL(m)$ with $1 \leq m < n$.

The formulation of the $GL(2)$ kernel over number fields was stimulated by two papers of Diaconu-Goldfeld (Gauss-Dirichlet conference and Bretton Woods), an older paper of A. Good (Selberg conference), remarks about spectral theory in some older papers of Sarnak, and a general conviction that many sensible structures can be profitably reformulated as having to do with spectral theory (representation theory) on adele groups. The $GL(2)$ case of this was joint work with Diaconu in spring 2006.

The eventual simplicity of the $GL(2)$ scenario suggested that *some* extension to larger groups was plausible. The ambient literature on Euler factorizations of global integrals, addressing Rankin-Selberg, Rankin-Shimura, and Langlands-Shahidi types, as well as Jacquet-Piatetski-Shapiro-Shalika's work on $GL(n)$, and Cogdell-Piatetski-Shapiro's work on converse theorems, and many others, provided a reassuring technical context.

More broadly, it is natural to speculate that global integrals (with explicable kernels) producing *integrals of* Euler products are conceivably useful.

Indeed, in hindsight, many features of the extension to $GL(n)$ are obvious, though non-trivial complications are apparent. In ongoing work, for various choices of defining data, we obtain asymptotics for moment integrals compatible with the convexity bound. Any better-power-of-T error term will break t-aspect convexity for a large class of $GL(m) \times GL(n)$ automorphic L–functions.
