TITLE: The p-adic Langlands conjecture for GL_2 over $\mathbf{Q}$.

ABSTRACT: The global p-adic Langlands conjecture for GL_2 over $\mathbf{Q}$ gives a (conjectural) description of the structure of the p-adically completed cohomology of the tower of classical modular curves. This global conjecture relies for its formulation on the existence of a local p-adic Langlands correspondence for GL_2 over Q_p. This local correspondence has been investigated extensively by Berger, Breuil, and Colmez.

In my talk I will first describe some of the recent progress on the construction of the local p-adic Langlands correspondence for GL_2 over Q_p. In particular, I will describe some work in progress of Colmez (which incorporates a deformation-theoretic strategy suggested by Kisin), the goal of which is to give a complete construction of the local correspondence.

I will then explain the statement of the global conjecture, and discuss some its applications (e.g. to the Fontaine-Mazur conjecture, and to the construction of families of p-adic L-functions). Finally, I will describe the (large number of) cases in which the conjecture can be proved, as well as other partial results in the direction of the conjecture.