Title: The weight part of Serre's conjecture over totally real fields

Abstract: Serre conjectured that all continuous, irreducible, odd $\rho : G_{\boldsymbol{Q}} \to GL_2(\overline{\boldsymbol{F}}_p)$ arise from modular forms. If ρ is modular, then proven refinements provide recipes for the possible weights and levels of the forms giving rise to it in terms of the local behavior of ρ. A natural generalization to the context of a totally real field F predicts that all continuous, irreducible, totally odd $\rho : G_F \to GL_2(\overline{\boldsymbol{F}}_p)$ arise from Hilbert modular forms. The recipe for the possible levels is similar to the case of $F = \boldsymbol{Q}$, but the (conjectural) recipe for the weights reveals features not so apparent for $F = \boldsymbol{Q}$. In particular, if ρ is locally reducible at a prime $\mathcal{P}$ over p, then the possible weights depend strongly on the corresponding extension of local characters. If ρ is locally semisimple at $\mathcal{P}$, then the recipe can be described in terms of the reduction of a corresponding irreducible characteristic zero representation of $GL_2(\mathcal{O}_F/\mathcal{P})$.