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Title: Heights on character varieties of reductive groups and a uniform Tits alternative

Abstract: We introduce a canonical height function on the character variety of a reductive group over the field of algebraic numbers. We establish an analogue of the classical Lehmer conjecture for this height function. Apart from group theoretical and geometrical ingredients, the proof makes key use of equidistribution properties of Galois orbits of algebraic points on tori. These diophantine geometric considerations lead to several applications about the structure of finitely generated subgroups of GL_n, including a uniform version of the Tits alternative. We show that given a subgroup G of $GL_n(K)$ generated by a finite set S of matrices, unless some finite index subgroup of G fixes a full flag, G must contain two independent elements a and b which are expressible as words in the elements of S of length uniformly bounded in terms of n only, independently of the field of definition K. This means that the subgroup generated by a and b is free. The result has several consequences for the geometry of Cayley graphs of infinite linear groups (growth, isoperimetry), but also for the structure (diameter and spectral gap) of Cayley graphs of finite simple groups of Lie type.