Speaker: Levent Alpoge

Title: Effective height bounds for odd-degree totally real points on some curves

Abstract: Let $\mathcal{O}$ be an order in a totally real field F. Let K be an odd-degree totally real field. Let S be a finite set of places of K. We study S-integral K-points on integral models $H_{\mathcal{O}}$ of Hilbert modular varieties because not only do said varieties admit complete curves (thus reducing questions about such curves' K-rational points to questions about S-integral K-points on these integral models), they also have their S-integral K-points controlled by known cases of modularity, in the following way. First assume for clarity modularity of all GL_2-type abelian varieties over K — then all S-integral K-points on $H_{\mathcal{O}}$ arise from K-isogeny factors of the $[F : \mathbb{Q}]$-th power of the Jacobian of a single Shimura curve with level structure (by Jacquet-Langlands transfer). By a generalization of an argument of von Känel, isogeny estimates of Raynaud/Masser-Wüstholz and Bost's lower bound on the Faltings height suffice to then bound the heights of all points in $H_{\mathcal{O}}(\mathcal{O}_{K,S})$. As for the assumption, though modularity is of course not known in this generality, by following Taylor's (sufficiently explicit for us) proof of his potential modularity theorem we are able to make the above unconditional.

Finally we use the hypergeometric abelian varieties associated to the arithmetic triangle group $\Delta(3,6,6)$ to give explicit examples of curves to which the above height bounds apply. Specifically, we prove that, for $a \in \overline{\mathbb{Q}}^{\times}$ totally real of odd degree (e.g. $a = 1$), for all $L/\mathbb{Q}(a)$ totally real of odd degree and S a finite set of places of L, there is an effectively computable $c = c_{a,L,S} \in \mathbb{Z}^+$ such that all $x, y \in L$ satisfying $x^6 + 4y^3 = a^2$ satisfy $h(x) < c$. Note that this gives infinitely many curves for each of which Faltings' theorem is now effective over infinitely many number fields.