SYLLABUS FOR GU4065 HONORS COMPLEX VARIABLES - FALL 2024

FRANCESCO LIN

Contacts. You can contact me at fl2550@columbia.edu. My office is Mathematics 613.

Hours and location. TuTh 11:40AM-12:55PM in Mathematics 520.

My office hours. TuTh 11-11:30AM and 1-1:30PM in my office Mathematics 613. Feel free to stop me for quick questions right after class. I will also most likely be in my office right before class.

Teaching assistants. Karina Dovgodko (undergraduate, kmd2235@columbia.edu, Help Room TuW 9-10AM) and Zoe Himwich (graduate, zmh2110@columbia.edu, Help Room M 6-7PM, W 3-4PM and 6-7PM).

Course Web Page. All class materials will be available on Coursework; all announcements will be made through the platform.

Prerequisites. Honors Mathematics A&B or Modern Analysis I. In particular, you are **expected** to have prior exposure to proof writing in real analysis. If that is not the case, but you still want to learn complex analysis, you are strongly encouraged to take UN3007 Complex Variables instead.

Textbook. I will post the handwritten notes I use in class after each lecture. No textbook is required, but good sources to look at are *Complex Analysis* by Lang, freely available on Springerlink, and *Complex Analysis* by Stein and Shakarchi.

Contents and goals. The main goal of the class is to learn the foundational results in the theory of holomorphic functions of a single variable, and their applications. Emphasis will be placed on *both* theoretical and computational aspects of the theory. We will also explore some interactions with neighboring subjects such as partial differential equations, differential geometry, algebraic topology and number theory. The rough outline of the class is the following:

(1) Holomorphic functions, the Cauchy-Riemann equations and harmonic functions
(2) (review of) metric spaces, uniform convergence, spaces of continuous functions
(3) Cauchy's theorem and its consequences.
(4) Singularities and the residue formula.
(5) Homotopy and winding numbers.
(6) Analytic continuation and the Gamma and Zeta functions.
(7) The prime number theorem.
(8) Conformal mappings and the Riemann mapping theorem.
(9) (time permitting) Introduction to hyperbolic geometry.

Assessment

Homework. There will be homework assignments (roughly) every week. You will find the contents and deadlines in the Assignments tab on the course web page. They will be due (via a file upload) on Thursdays at 11PM.

If you plan on turning in handwritten solutions, make sure to upload a **single, readable PDF**. Documents that do not satisfy this requirement will not be taken into consideration. Your solutions should be clearly marked with problem numbers. Your name (first and last) needs to be written clearly and legibly at the beginning of each set. The graders will be instructed to take away points for anything that causes them undue difficulty in grading your homework, including poor presentation, organization, or handwriting.

The course policy is that **homework delays are not accepted**. However, your two lowest homework scores will not count towards your final grade. Use this for unexpected circumstances such as illnesses when you are unable to do or hand in your assignment on time. Do the problems later, on your own, so that you learn the material.

Homework is the best way to stay up to date with the material. You are **encouraged** to collaborate with your classmates, but you should hand in your own copy of the assignment, with solutions written in your own words. Identical copies are a violation of the expected standard of academic integrity and will be dealt with according to university policy.

Exams. The exams will contain both computational problems and proof-based questions. There will be a 75 minutes in-class Midterm on Tuesday October 22nd. The final exam date TBA is scheduled by the Registrar, and it will cover the entire course.

There will not be make-up exams. You must plan to take the midterm and final exams at the scheduled time. Besides students with disabilities having prior arrangements with ODS, the only exceptions will be for those with an incapacitating illness, a serious family emergency, or situations of comparable gravity. In both cases you will need a note from your advising dean. Incompletes can be granted only by your advising dean and only in the circumstances mentioned above.

Books, notes, calculators, and other electronic devices will not be allowed (or needed) on quizzes or exams. Anyone guilty of academic dishonesty, such as cheating on an exam or helping someone else to cheat, will automatically fail the course and faces further academic discipline.

Students with disabilities. In order to receive disability-related academic accommodations for this course, students must first be registered with their school Disability Services (DS) office.

Exam preparation. I will provide a problem sheet roughly one week before the actual exams; these will be close in spirit to the actual test, and it is recommended for you to work on them on your own in a test-like setup. I will provide sketches of solutions.

Grading. Your final score will be computed via the following weights:

- Homework: 20%
- Midterm: 35%
- Final: 45%