Down the rabbit hole

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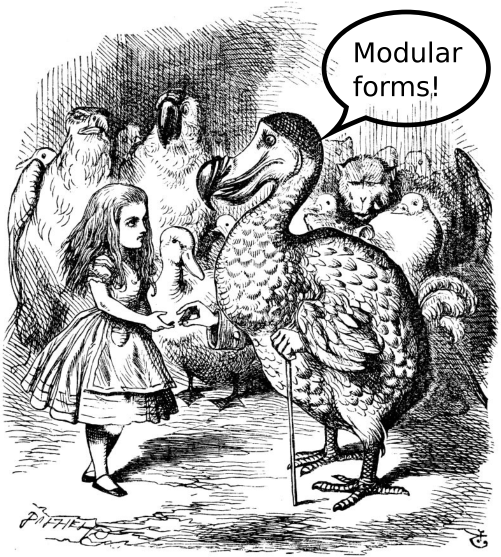


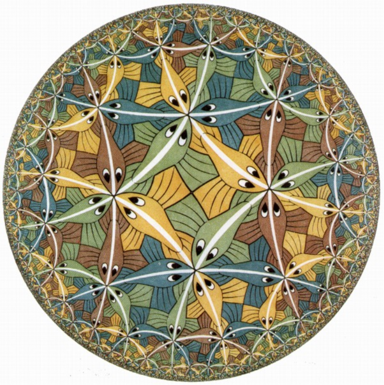
Symmetries and complex numbers

M. C. Escher, drawing E55

- Rotations and translations in the complex plane are described by functions of the form

$$f(z) = e^{i\theta} z + c\,.$$

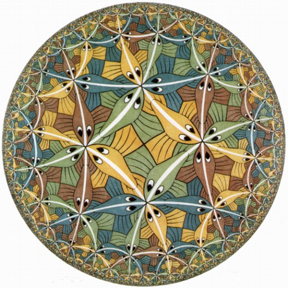
Symmetries and complex numbers



M. C. Escher, Circle Limit III

- How to describe the symmetries of this drawing with complex numbers?

Symmetries and complex numbers



- How to describe the symmetries of this drawing with complex numbers?

$$f(z) = \frac{az+b}{b^*z+a^*}, \quad a, b \in \mathbb{C}, \quad |a| > |b|$$

- Why these transformations? They preserve the unit circle: if $|z| = 1$, then

$$|az+b| = |azz^* + bz^*| = |a + bz^*| = |a^* + b^*z|$$

so $\frac{|az+b|}{|b^*z+a^*|} = 1$.

Symmetries and complex numbers

J. Leys, after M. C. Escher, drawing E69

- How to describe the symmetries of this drawing with complex numbers?

$$f(z) = \frac{az+b}{cz+d}, \quad a, b, c, d \in \mathbb{R}, \quad ad - bc > 0$$

- These transformations preserve the upper half plane.

Thanks for attending!