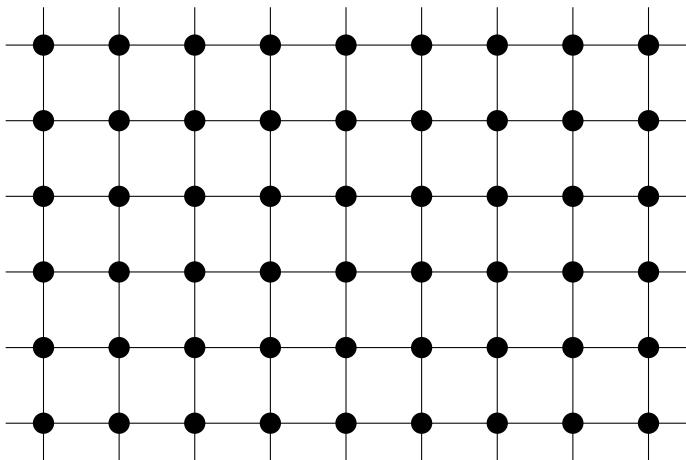


Suppose you are designing a computer network.

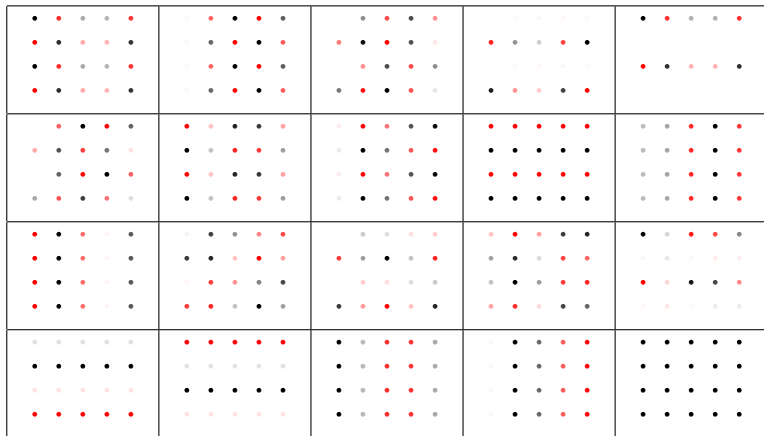
- ▶ You'd like to be able send information quickly between any pair of computers.
- ▶ Due to budget constraints, you can only connect each computer directly to 4 others.
- ▶ For simplicity, assume that it doesn't matter where the computers are physically located, only which ones are connected to each other.

One possible network:

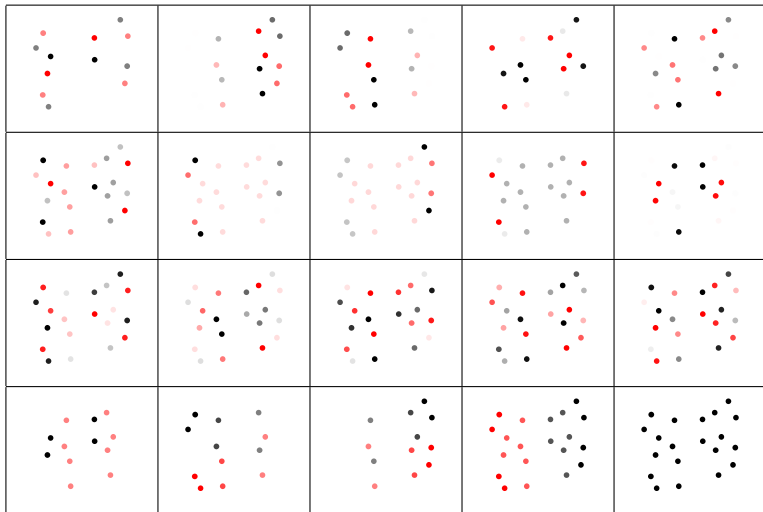


1. How would you quantify how good a network is?
2. How would you design a network that is better than the one above?

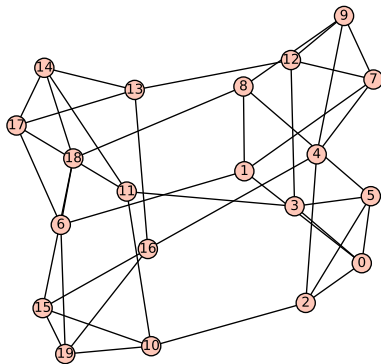
Eigenvectors of the adjacency matrix



Eigenvectors of the adjacency matrix

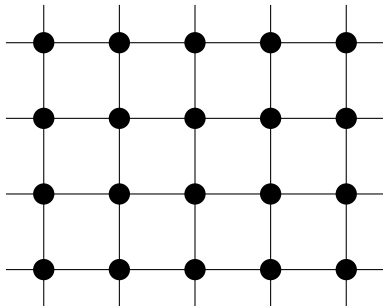


A Ramanujan graph



- ▶ This graph is 4-regular.
- ▶ The nontrivial eigenvalues of its adjacency matrix have absolute value at most $3 \leq 2\sqrt{4-1} = 2\sqrt{3} \approx 3.46$.

A graph that is not Ramanujan



- ▶ One the eigenvalues of its adjacency matrix is $-\frac{5+\sqrt{5}}{4} \approx -3.62 < -2\sqrt{3} \approx -3.46$.