1. a) Write down a sum which estimates the area under the curve $\sin(x)$ between $x = 0$ and $x = \pi$.
b) Write down a limit which is exactly equal to the area under the curve $\sin(x)$ between $x = 0$ and $x = \pi$, and then evaluate that limit. You will need to know that $\sum_{k=0}^{n} \sin \left(\frac{k \pi}{n}\right) = \cot \left(\frac{\pi}{2n}\right)$.

2. The speed of a ball is given by $v(t) = \frac{1}{4} t^2$ (so for example, at $t = 0$ s, the ball is not moving, while at $t = 1$ s, the ball is moving at $\frac{1}{4}$ m s$^{-2}$). At $t = \frac{1}{2}$ s, the ball is moving at $\frac{1}{4}$ m s$^{-2}$. How far does the ball travel between $t = 0$ s and $t = 1$ s? It will be useful to know that $\sum_{j=0}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}$.

3. a) Estimate the length of the curve $y = x^2$ between $x = 0$ and $x = 1$. For example, a first approximation might be that the length of the curve is just the distance between $(0, 0)$ and $(1, 1)$, which is $\sqrt{2}$. If you draw lines between $(0, 0)$ and $\left(\frac{1}{2}, \frac{1}{4}\right)$, and then between $\left(\frac{1}{2}, \frac{1}{4}\right)$ and $(1, 1)$, and take the length of those two lines, you get a better approximation of the length of the curve.
b) Write down a limit whose value is the exact length of the curve $y = x^2$ between $x = 0$ and $x = 1$. This limit will be hard to evaluate (and you don’t have to evaluate it), but write down exactly what it is.

4. What are the antiderivatives of the following functions?
a) $0$
b) $1$
c) $x$
d) $\frac{1}{\sqrt{x}}$
e) $ax^n$
f) $\frac{1}{x}$
g) $\sin(x)$
h) $\sin(x) \cos(x)$
i) $e^{ax}$
j) $2x e^{x^2}$
k) $\frac{x}{\sqrt{x^2 + 1}}$
l) $f(x) - x f'(x)$
m) $\log(x)$