Calculus 1 Assignment 5

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Due Thursday, October 29th at 5 pm

1. A particle is moving along the hyperbola $xy = 8$. As it reaches the point $(4, 2)$, the $y$ coordinate is decreasing at a rate of $3 \text{cm/s}$. How quickly is the $x$ coordinate changing at that instant?

2. 
a) What is the derivative of $e^t$ at $t = 0$?
b) What is the derivative of $\arctan(x)$ at $x = 1$?
c) State the chain rule for $f(g(x))$ in both Newton notation and Leibniz notation.
d) Estimate $\arctan(e^{0.1})$. See if you can do it in your head before writing things down (but after that do it while writing things down).

3. 
a) What is the best linear approximation to $\log(\alpha x + 1)$ at $x = 0$? ($\alpha$ is the greek letter “alpha”. It’s just a fixed but arbitrary constant here)
b) Use this linear approximation to show that
\[
\lim_{x \to 0} \frac{\log(\alpha x + 1)}{x} = \alpha
\]
Be sure to justify your work.

4. 
a) Use linear approximation to estimate $\sqrt{9.01}$. How far off are you? (Consult a calculator to answer the second part.)
b) Use linear approximation to estimate $\sqrt{10}$. Do you think this approximation is an overestimate or an underestimate? Try to guess how far off you are (write down a number).
After answering the questions above (and not before), use a calculator to calculate $\sqrt{10}$.

5. 
a) Show that $\sqrt{2x^6 + 3x^2 - 4} = \sqrt{2x^3} + O(x)$. What is the analogous statement for $(3x^9 + 4x^8 - x^2 - 100)^{\frac{1}{5}}$?
b) Show that
\[
\lim_{x \to \infty} \frac{\sqrt{2x^6 + 3x^2 - 4}}{x^3} = \sqrt{2}
\]
c) Evaluate
\[
\lim_{x \to \infty} \frac{(3x^9 + 4x^8 - x^2 - 100)^{\frac{1}{5}}}{x^r}
\]
(Your answer should be a function of $r$.)