CALCULUS III SPRING 2008
MIDTERM I

(1) Consider the four vectors $\vec{u} = \langle 2, -6, 4\rangle$, $\vec{v} = \langle 1, 1, 1\rangle$, $\vec{w} = \langle -1, -2, 3\rangle$ and $\vec{t} = \langle -1, 3, -2\rangle$.
(i) Which of them is the shortest, the longest? Find their lengths.
(ii) Which of the four vectors are parallel to each other?

(2) Let $\vec{u}$ be a vector in the space.
(i) Suppose that for each vector $\vec{v}$ in the space, the dot product $\vec{u} \cdot \vec{v} = 0$. Show that $\vec{u} = \vec{0}$ in two different ways: one using coordinates and one without.
(ii) Suppose that for each vector $\vec{v}$ in the space, the cross product $\vec{u} \times \vec{v} = \vec{0}$. Show that $\vec{u} = \vec{0}$ in two different ways: one using coordinates and one without.

(3) Consider the three points $A = (1, 2, 3)$, $B = (2, 4, 3)$ and $C = (0, -1, -5)$.
(i) Find an equation of the plane Π that contains A, B and C.
(ii) Find parametric equations for the line L through A and $D = (0, 0, 1)$.
(iii) Find an equation of the sphere S whose a diameter is $[AB]$.
(iv) Find the intersection of L and Π. Find the intersection of L and S.

(4) Show that the surface with equation in spherical coordinates (ρ, θ, ϕ):

$$\rho = 4 \sin\phi \cos(\theta - \frac{\pi}{4})$$

is a sphere. Give its radius and the rectangular coordinates of its center.

(5) A surface consists of all points P such that the distance from P to the plane $x = 1$ is half the distance from P to the point $(0, -1, 1)$. Find an equation for this surface and identify it.

(6) Let $\vec{u}$, $\vec{v}$, $\vec{w}$ and $\vec{t}$ be four vectors in the space. Prove using coordinates that:

$$(\vec{u} \times \vec{v}) \cdot (\vec{w} \times \vec{t}) = \begin{vmatrix} \vec{u} \cdot \vec{w} & \vec{v} \cdot \vec{w} \\ \vec{u} \cdot \vec{t} & \vec{v} \cdot \vec{t} \end{vmatrix}$$

where the right hand side is the determinant of order 2.