Calculus I
Review Sheet 1

1 Concept Summary

The following concepts and skills from chapters 2 and 3 were covered in class and may be tested on the exam:

1. Limits
   (a) Definition of a limit
   (b) Definition of a one-sided limit
   (c) Estimating value of a limit numerically and graphically
   (d) Cases when limits do not exist
      i. Left-hand limit is not equal to the right-hand limit
      ii. Infinite oscillation
      iii. Infinite limits, vertical asymptotes
   (e) Special limit $\lim_{x\to 0} \frac{\sin x}{x} = 1$
   (f) Using limit laws to compute limits algebraically (read the smallprint on the laws)
   (g) Using the Squeeze Theorem to compute limits
   (h) Applications of limits
      i. Tangent lines
      ii. Instantaneous velocity
2. Continuity
   (a) Definition of continuity at a point
   (b) Definition of one-sided continuity
   (c) Continuity on an interval
   (d) Continuity of combinations of functions
   (e) Functions that are continuous at every number of the domain
   (f) Using continuity of f to evaluate limits
      i. $\lim_{x\to a} f(x) = f(a)$
      ii. $\lim_{x\to a} f(g(x)) = f(\lim_{x\to a} g(x))$
   (g) The Intermediate Value Theorem
      i. Precise statement of the IVT
      ii. Using IVT to prove the existence of a root of an equation
3. Derivatives
   (a) Definition of a derivative at a point
   (b) Derivative as a function
   (c) Differentiable functions
      i. Intervals of differentiability of a function
      ii. Why a function might fail to be differentiable
   (d) Higher derivatives
4. Computing derivatives
   (a) Directly from the definition
   (b) Using formulas for derivatives of power, exponential, logarithmic, trig, and inverse trig functions
   (c) Using the sum, difference, product, quotient, and chain rules
   (d) Using implicit differentiation
   (e) Using logarithmic differentiation

2 Practice Exercises

The following exercises from the book will be helpful:

- p.176, Concept Check (all questions, but ignore the part where they ask about horizontal asymptotes)
- p.176, True-False Quiz (omit 9, 10, and 15)
- p.177-179, exercises: 1-22, 25-26, 31-39, 44-46.
- p.270, Concept Check (omit 2oqrt, 3b, and 6)
- p.270, True-False Quiz
- p.271-273, exercises: 1-46 (omit 43 and 45), 49-51, 54-59, 64, 65, 67-79, 81-83, 100-102, 104.

3 Sample exam questions

1. Evaluate each of the following limits or explain why it does not exist. If a limit is $+\infty$ or $-\infty$, state this as your answer:
   (a) $\lim_{t\to 2^+} \dfrac{t^2+t-6}{3t^2-5t-2}$
   (b) $\lim_{x\to -5} \dfrac{|x+5|}{x+5}$
   (c) $\lim_{t\to 0} \left(\dfrac{1}{3t} - \dfrac{1}{t\sqrt{9+t}}\right)$
   (d) $\lim_{x\to -1^-} \dfrac{x^2+1}{x^2-1}$
   (e) $\lim_{x\to 0} |x| \cos\dfrac{1}{x}$ (Hint: use the Squeeze Theorem)
2. Roughly sketch the graph of an example of a function f that satisfies *all* of the following conditions:
   (a) $\lim_{x\to 0^+} f(x) = 10$
   (b) $\lim_{x\to 0^-} f(x) = \infty$
   (c) $f(0) = 0$
   (d) $\lim_{x\to 5} f(x) = 9$
   (e) f is continuous on $(0, +\infty)$
   (f) $\lim_{x\to -3^-} f(x) = 0$
   (g) $\lim_{x\to -3^+} f(x) = 3$
   (h) f is continuous on $(-\infty, -3]$
3. Let
$$f(x) = \begin{cases} \frac{1}{x^2+5x} & \text{if} \quad x < 0 \\ x^2 + c & \text{if} \quad 0 \le x \le 1 \\ cx + 5 & \text{if} \quad x > 1 \end{cases}.$$
   (a) Carefully state the definition that a function f is continuous at a point $x = a$.
   (b) Evaluate each limit, if it exists:
      i. $\lim_{x\to 0^+} f(x)$
      ii. $\lim_{x\to 0^-} f(x)$
      iii. $\lim_{x\to -5^+} f(x)$
      iv. $\lim_{x\to -5^-} f(x)$
   (c) Find the value of c that makes $f(x)$ continuous at 1.
   (d) Where is f discontinuous?
   (e) Find the equations of vertical asymptotes.
4. Determine the interval on which each of the following functions is continuous. (Hint: explain, using the appropriate results about continuity, why these functions are continuous at every number of their domain and state the domain.)
   (a) $f(x) = 5x^3 + 3x^2 + \sqrt{x} + 2$
   (b) $g(x) = \frac{1}{\sqrt{1-x^2}}$
   (c) $h(x) = \cos(\sin x)$
5. (a) Carefully state the Intermediate Value Theorem.
   (b) Would you be willing to bet that the equation
$$x^3 + 12x^2 + x - 10 = 0$$
   has a solution on the interval $(0, 1)$? Explain why or why not.
   (c) What about the equation
$$\frac{2x^2 - 3x + 1}{2x - 1} = 0,$$
   would you be willing to bet that it has a solution on the interval $(0, 1)$? Explain why or why not.
6. Let $f(x) = \begin{cases} x^3 + x & \text{if} \quad x < 3 \\ cx + 15 & \text{if} \quad x \ge 3 \end{cases}$, where c is some fixed real number
   (a) Find a value of c such that $f(x)$ is continuous on $(-\infty, \infty)$.
   (b) Will that value of c make $f(x)$ differentiable on $(-\infty, \infty)$? Explain.
7. Let $f(x) = \dfrac{4}{5-2x}$. Find $f'(x)$ using *only* the definition of derivative.
8. In each case, find the derivative, $\dfrac{dy}{dx}$. Do *not* simplify:
   (a) $y = \csc 5x + 3\tan x$
   (b) $y = \dfrac{x^7}{5} + \dfrac{5}{x^7} - \pi^4$
   (c) $y = \dfrac{\sqrt{5x^3 - 7x^5}}{\cos x^2}$
   (d) $y = (x^5 - 4x^3 + 5)\left(\dfrac{3}{x^3} - 5\sqrt{x}\right)$
   (e) $y = \csc(\sin\sqrt[7]{27 - 5x^2})$
   (f) $y = (x^2+1)^{\sin 3x}$
9. Find an equation of the tangent line to the curve $xy^4 + x^2y = 14$ at the point $(1, -2)$.

4 Answers

1. (a) $\frac{5}{7}$
   (b) does not exist (one-sided limits not equal)
   (c) $\frac{1}{54}$
   (d) $+\infty$
   (e) 0
2. answers may vary
3. (a) A function $f(x)$ is said to be continuous at $x = a$ if $\lim_{x\to a} f(x) = f(a)$
   (b) c, $-\infty$, $-\infty$, $+\infty$
   (c) No such c exists

(d) At -5, 0, and 1

(e) $x = 0$ and $x = -5$

4. (a) $[0, \infty)$

   (b) $(-1, 1)$

   (c) $(-\infty, \infty)$

5. (a) Let a function $f(x)$ be continuous on $[a, b]$ and let N be a number such that $f(a) \leq N \leq f(b)$ (or $f(b) \leq N \leq f(a)$). Then there exists a number $c \in (a, b)$ such that $f(c) = N$.

   (b) Yes. Let $f(x) = x^3 + 12x^2 + x - 10$. It is continuous on $[0, 1]$, $f(0) < 0$, $f(1) > 0$, so by the IVT there is a number, c, between 0 and 1 such that $f(c) = 0$.

   (c) I won't bet on it without actually trying to solve the equation first. We can't apply the IVT because the function is not continuous on the interval $[0, 1]$

6. $c = 5$ makes $f(x)$ continuous everywhere, but $f(x)$ is not differentiable at 3

7. Use $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

8. See derivatives.pdf

9. $y_t = \frac{12}{31}(x - 1) - 2$