

1. Compute each of the following limits. If a limit does not exist, explain why. If a limit is ∞ or $-\infty$, state that as your answer.

(a) (3 points) $\lim_{x \rightarrow -\infty} \frac{\sqrt{4x^4 - x}}{x^2 + 1}$

(b) (3 points) $\lim_{x \rightarrow 1} \frac{|x - 1|}{x - 1}$

(c) (3 points) $\lim_{x \rightarrow 0} \frac{\ln(\cos x)}{x}$

(d) (3 points) $\lim_{x \rightarrow \infty} \frac{1}{x} \sin x$
(Hint: $-1 \leq \sin x \leq 1$)

2. Find the derivative, $f'(x)$, of the following functions $f(x)$ (simplification is unnecessary):

(a) (3 points) $f(x) = \tan^2(2x) + \frac{1}{x^2} + e^\pi$

(b) (3 points) $f(x) = \frac{e^x}{1+x^2}$

(c) (3 points) $f(x) = (x^2 - x + 1)^x$

(d) (3 points) $f(x) = \int_x^1 \sin t dt$

3. (5 points) Use differentials (linear approximation) to estimate $\sqrt{99.8}$.

4. (5 points) Find an equation of the tangent line to the curve $xy^3 + x^3y = 10$ at the point $(1, 2)$.

5. (6 points) At 10 am, a train leaves the station traveling north at 80 mph. At 11 am, a second train leaves the same station traveling east at 90 mph. Determine how far apart the trains are at 1 pm. At what rate is their distance changing at 1 pm?

6. (6 points) Find the dimensions of a rectangle with area 100 m^2 whose perimeter is as small as possible.

7. (6 points) Find the absolute maximum and absolute minimum values of $f(x) = (x^2 - 1)^3$ on the interval $[-1, 2]$.

8. (6 points) How many solutions does the equation $3x = \sin x$ have? Explain.

9. A particle is moving in a straight line with velocity $v(t) = 3t^2 - 2t + 1$.

(a) (2 points) Find $a(t)$, that is, the acceleration of the particle at time t .

(b) (3 points) Find $s(t)$, that is, the position of the particle at time t , given that the position at time $t = 1$ is 3.

10. (5 points) Find the average value of $f(x) = \sqrt{1-x}$ for $\frac{3}{4} \leq x \leq 1$.

11. Evaluate each of the following:

(a) (3 points) $\int_0^1 \sqrt{1-x^2} dx$
(Hint: sketch $y = \sqrt{1-x^2}$)

(b) (4 points) $\int \frac{x^2 + 1}{\sqrt{x}} dx$

(c) (4 points) $\int \frac{x}{1+x^4} dx$

12. Let R be the region bounded by $y = \sqrt{x}$ and $y = x$.

(a) (3 points) Find the area of R .

(b) (2 points) Find the volume of the solid obtained by spinning R around the x -axis.

13. (16 points) Sketch the graph of $f(x) = x \ln x$. Show all work needed to justify end behavior, intervals of increase and decrease, concavity, extrema and inflection points, vertical and horizontal asymptotes (if any).