1. Roughly sketch the graph of an example of a function f that satisfies all of the following conditions:

   (a) $\lim_{x\to 0^+} f(x) = 3$

   (b) $\lim_{x\to 0^-} f(x) = -\infty$

   (c) $f(0) = 0$

   (d) f is continuous on $(0, +\infty)$

   (e) $\lim_{x\to -2^+} f(x) = \infty$

   (f) f is continuous on $(-\infty, -2]$

   (g) f is not differentiable at 3

2. Compute each of the following limits. If the limit does not exist, explain why. If the limit is ∞ or $-\infty$, state that as your answer.

   (a) $\lim_{x\to 7} \sqrt{x+2} =$

   (b) $\lim_{x\to -1} \frac{|x+1|}{x^2+2} =$

   (c) $\lim_{x\to \frac{\pi}{2}^-} \tan x =$

   (d) $\lim_{t\to 9} \frac{9-t}{3-\sqrt{t}} =$

   (e) $\lim_{x\to 0} \frac{\sin 3x}{2x} =$

   (f) (Hint: recall the definition of derivative)
   $\lim_{h\to 0} \frac{e^{\cos h} - e}{h} =$

3. Let $f(x)$ be a function such that $\lim_{x\to 2} f(x) = 3$.

   (a) Can it happen that $f(1.999999) = 1700$? Explain.

(b) Can it happen that $f(2) = -23$? Explain.

(c) How will your answers in parts (a) and (b) change if we also know that $f(x)$ is continuous at $x = 2$?

4. In each case, find the derivative, y':

   (a) $y = x^4 + \sqrt[3]{x^2} + e^{-3}$

   (b) $y = \dfrac{3x - 2}{\sqrt{2x + 1}}$

   (c) $y = \tan\left(\arcsin\left(\sqrt{x}\right)\right)$

   (d) $y = (x^x + 1)(\ln(3x + 1) - \tan 5x^2)$

5. The following (incorrect) argument is given to show that $\lim_{x \to 0} x^4 \sin \dfrac{1}{x} = 0$. Find the error in that argument, and give a correct argument below:

$$\begin{aligned} \lim_{x \to 0} x^4 \sin \frac{1}{x} &= \lim_{x \to 0} x^4 \cdot \lim_{x \to 0} \sin \frac{1}{x} \\ &= 0 \cdot \lim_{x \to 0} \sin \frac{1}{x} \\ &= 0 \end{aligned}$$

6. Prove the power rule, that is, prove that $(x^a)' = ax^{a-1}$ for any real number a. (Hint: logarithmic differentiation)

7. Determine whether the following statements are true or false. Briefly justify your choice.

(a) There are functions $f(x)$ and $g(x)$ such that neither $\lim_{x \to a} f(x)$ nor $\lim_{x \to a} g(x)$ exist, yet $\lim_{x \to a} [f(x) - g(x)]$ exists.

(b) Let $f(x)$ be a continuous function. Suppose that $f(12) > 0$, and $f(x) = 0$ if and only if $x = -1$, $x = 0$, or $x = 3$. Then $f(100) > 0$.

(c) If $x = 1$ is a vertical asymptote of $y = f(x)$ then $f(1)$ is not defined.