Exam Title: CALCULUS I FINAL Name:______________________________
Instructor: A. Kontorovich
Date: December 20th, 2004, 7:10 - 10:00 PM.

You will have three hours to complete this exam. No books, notes, or calculators of any kind. Please show ALL work for full credit. Take a deep breath. Relax. Good luck!

1) Compute $\lim_{x \to 2} \dfrac{\sqrt{x+2} - \sqrt{2x}}{x^2 - 2x}$.

2) Suppose that $|f(x)| \leq g(x)$ for all x, and that $\lim_{x \to a} g(x) = 0$. Compute $\lim_{x \to a} f(x)$.

3) Let $f(x) = \begin{cases} 2x - x^2, & 0 \le x < 2 \\ 2 - x, & 2 \le x < 3 \\ x - 4, & 3 \le x < 4 \\ \pi, & x \ge 4 \end{cases}$. State the domain of f. For $a = 2, 3,$ and 4, discover whether f is continuous at a. For each discontinuity, classify it. Sketch a graph of f. State the range of f.

4) Evaluate $\lim_{t \to 0} \dfrac{t^3}{\tan^3(2t)}$.

5) Use the definition of derivative to find $f'(2)$ for $f(x) = x^3 - 2x$. Find the equation of the tangent line to this curve at the point $(2, 4)$.

6) Calculate $y' = \frac{dy}{dx}$.

a) $y = x^r e^{sx}$.

b) $y = \ln(x^2 e^x)$.

c) $xe^y = y - 1$.

7) Find $f^{(n)}(x)$ for $f(x) = \frac{1}{2-x}$.

8) By differentiating the double-angle formula for cosine, obtain the double-angle formula for sine.

9) Use linearization to approximate $\sqrt[3]{1.03}$.

10) The volume of a cube is increasing at a rate of 10 cm^3/min. How fast is the surface area increasing when the length of an edge is 30 cm.

11) Evaluate $\lim_{x \to (\pi/2)^-} (\tan x)^{\cos x}$.

12) Find the local and absolute extreme values of the function $f(x) = \dfrac{x}{x^2 + x + 1}$ on the interval $[-2, 0]$.

13) Use the guidelines to sketch the curve $y = \frac{x^2}{x+8}$. Label all local extrema, intercepts, and points of inflection.

14) Show that the function $f(x) = x^{101} + x^{51} + x - 1$ has exactly one real root.

15) Find two positive integers such that the sum of the first number and four times the second number is 1000 and the product of the numbers is as large as possible.

16) Find f if $f''(x) = 20x^3 + 24x^2 - 6x + 6$, $f(0) = 1$ and $f(1) = 0$.

17) $\int \frac{\cos(\ln x)}{x} dx$

18) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

19) Find $\lim_{h \to 0} \frac{1}{h} \int_{2}^{2+h} \sqrt{1 + t^3}\, dt$.

20) Evaluate $\lim_{n \to \infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^9 + \left(\frac{2}{n}\right)^9 + \ldots + \left(\frac{n}{n}\right)^9\right]$.

21) Find the area of the region bounded by $x + y = 0$ and $x = y^2 + 3y$.

22) The base of a solid is the region bounded by the parabolas $y = x^2$ and $y = 2 - x^2$. Find the volume of the solid if the cross-sections perpendicular to the x-axis are squares with one side lying along the base.

23) Find the volume of the solid obtained by rotating the region bounded by the curves $y = x$ and $y = x^2$ about the line $y = 2$.

24) Find the average value of the function $f(t) = t\sin(t^2)$ on the interval $[0, 10]$.

Extra Credit) By differentiating both sides, prove that if f is continuous, then $\int_0^x f(u)(x - u)du = \int_0^x \left(\int_0^u f(t)dt\right)du$.