3. \( x = \cos^2 t, \; y = 1 - \sin t, \; 0 \leq t \leq \pi / 2 \)

<table>
<thead>
<tr>
<th>( t )</th>
<th>0</th>
<th>( \pi / 6 )</th>
<th>( \pi / 3 )</th>
<th>( \pi / 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>1</td>
<td>( 3 / 4 )</td>
<td>( 1 / 4 )</td>
<td>0</td>
</tr>
<tr>
<td>( y )</td>
<td>1</td>
<td>( 1 / 2 )</td>
<td>( 1 - \frac{\sqrt{3}}{2} \approx 0.13 )</td>
<td>0</td>
</tr>
</tbody>
</table>

4. \( x = e^{-t} + t, \; y = e^t - t, \; -2 \leq t \leq 2 \)

<table>
<thead>
<tr>
<th>( t )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>( e^2 - 2 )</td>
<td>( e - 1 )</td>
<td>1</td>
<td>( e^{-1} + 1 )</td>
<td>( e^{-2} + 2 )</td>
</tr>
<tr>
<td></td>
<td>5.39</td>
<td>1.72</td>
<td>1.37</td>
<td>2.14</td>
<td></td>
</tr>
<tr>
<td>( y )</td>
<td>( e^{-2} + 2 )</td>
<td>( e^{-1} + 1 )</td>
<td>1</td>
<td>( e - 1 )</td>
<td>( e^2 - 2 )</td>
</tr>
<tr>
<td></td>
<td>2.14</td>
<td>1.37</td>
<td>1.72</td>
<td>5.39</td>
<td></td>
</tr>
</tbody>
</table>

6. \( x = 1 - 2t, \; y = \frac{1}{2} t - 1, \; -2 \leq t \leq 4 \)

(a)

<table>
<thead>
<tr>
<th>( t )</th>
<th>-2</th>
<th>0</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>5</td>
<td>1</td>
<td>-3</td>
<td>-7</td>
</tr>
<tr>
<td>( y )</td>
<td>-2</td>
<td>-1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

(b) \( x = 1 - 2t \Rightarrow 2t = -x + 1 \Rightarrow t = -\frac{1}{2}x + \frac{1}{2} \), so

\[ y = \frac{1}{2}t - 1 = \frac{1}{2}(-\frac{1}{2}x + \frac{1}{2}) - 1 = -\frac{1}{4}x + \frac{1}{4} - 1 \Rightarrow y = -\frac{1}{4}x - \frac{3}{4} \]

with \(-7 \leq x \leq 5\)

8. \( x = t - 1, \; y = t^2 + 1, \; -2 \leq t \leq 2 \)

(a)

<table>
<thead>
<tr>
<th>( t )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>-3</td>
<td>-2</td>
<td>-1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>( y )</td>
<td>-7</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>9</td>
</tr>
</tbody>
</table>

(b) \( x = t - 1 \Rightarrow t = x + 1 \Rightarrow y = (x + 1)^2 + 1 \),

or \( y = x^2 + 3x^2 + 3x + 2 \), with \(-3 \leq x \leq 1\)
10. $x = t^2, y = t^3$

(a) 

<table>
<thead>
<tr>
<th>$t$</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>$y$</td>
<td>8</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>8</td>
</tr>
</tbody>
</table>

(b) $y = t^3 \Rightarrow t = \sqrt[3]{y} \Rightarrow x = t^2 = (\sqrt[3]{y})^2 = y^{2/3}$. $t \in \mathbb{R}, y \in \mathbb{R}, x \geq 0$.

14. (a) $x = e^t - 1, y = e^{2t}$.

$y = (e^t)^2 = (x + 1)^2$ and since $x > -1$, we have the right side of the parabola $y = (x + 1)^2$.

15. (a) $x = e^{2t} \Rightarrow 2t = \ln x \Rightarrow t = \frac{1}{2} \ln x$.

$y = t + 1 = \frac{1}{2} \ln x + 1$.

16. (a) $x = \sqrt{t + 1} \Rightarrow x^2 = t + 1 \Rightarrow t = x^2 - 1$.

$y = \sqrt{1 - 1} = \sqrt{(x^2 - 1) - 1} = \sqrt{x^2 - 2}$. The curve is the part of the hyperbola $x^2 - y^2 = 2$ with $x \geq \sqrt{2}$ and $y \geq 0$.

18. (a) $x = \tan^2 \theta, y = \sec \theta, -\pi/2 < \theta < \pi/2$.

$1 + \tan^2 \theta = \sec^2 \theta \Rightarrow 1 + x = y^2 \Rightarrow x = y^2 - 1$. For $-\pi/2 < \theta \leq 0$, we have $x \geq 0$ and $y \geq 1$. For $0 < \theta < \pi/2$, we have $0 < x$ and $1 < y$. Thus, the curve is the portion of the parabola $x = y^2 - 1$ in the first quadrant. As $\theta$ increases from $-\pi/2$ to 0, the point $(x, y)$ approaches $(0, 1)$ along the parabola. As $\theta$ increases from 0 to $\pi/2$, the point $(x, y)$ retreats from $(0, 1)$ along the parabola.
24. (a) From the first graph, we have $1 \leq x \leq 2$. From the second graph, we have $-1 \leq y \leq 1$. The only choice that satisfies either of those conditions is II.

(b) From the first graph, the values of $x$ cycle through the values from $-2$ to $2$ four times. From the second graph, the values of $y$ cycle through the values from $-2$ to $2$ six times. Choice I satisfies these conditions.

(c) From the first graph, the values of $x$ cycle through the values from $-2$ to $2$ three times. From the second graph, we have $0 \leq y \leq 2$. Choice IV satisfies these conditions.

(d) From the first graph, the values of $x$ cycle through the values from $-2$ to $2$ two times. From the second graph, the values of $y$ do the same thing. Choice II satisfies these conditions.

38. (a) $x = t$, so $y = t^{-2} = x^{-2}$. We get the entire curve $y = 1/x^2$ traversed in a left-to-right direction.

(b) $x = \cos t$, $y = \sec^2 t = \frac{1}{\cos^2 t} = \frac{1}{x^2}$. Since $\sec t \geq 1$, we only get the parts of the curve $y = 1/x^2$ with $y \geq 1$. We get the first quadrant portion of the curve when $x > 0$, that is, $\cos t > 0$, and we get the second quadrant portion of the curve when $x < 0$, that is, $\cos t < 0$.

(c) $x = e^t$, $y = e^{-2t} = (e^t)^{-2} = x^{-2}$. Since $e^t$ and $e^{-2t}$ are both positive, we only get the first quadrant portion of the curve $y = 1/x^2$.

40. The first two diagrams depict the case $\pi < \theta < \frac{3\pi}{2}$. As in Example 7, $C$ has coordinates $(r \theta, r)$. Now $Q$ (in the second diagram) has coordinates $(r \theta, r + d \cos(\theta - \pi)) = (r \theta, r - d \cos \theta)$, so a typical point $P$ of the trochoid has coordinates $(r \theta + d \sin(\theta - \pi), r - d \cos \theta)$. That is, $P$ has coordinates $(x, y)$, where $x = r \theta - d \sin \theta$ and $y = r - d \cos \theta$. When $d = r$, these equations agree with those of the cycloid.
42. A has coordinates \((a \cos \theta, a \sin \theta)\). Since \(OA\) is perpendicular to \(AB\), \(\Delta OAB\) is a right triangle and \(B\) has coordinates \((a \sec \theta, 0)\). It follows that \(P\) has coordinates \((a \sec \theta, b \sin \theta)\). Thus, the parametric equations are \(x = a \sec \theta, y = b \sin \theta\).

43. \(C = (2a \cot \theta, 2a)\), so the \(x\)-coordinate of \(P\) is \(x = 2a \cot \theta\). Let \(B = (0, 2a)\).

Then \(\angle OAB\) is a right angle and \(\angle OBA = \theta\), so \(|OA| = 2a \sin \theta\) and 
\[A = ((2a \sin \theta) \cos \theta, (2a \sin \theta) \sin \theta)\]. Thus, the \(y\)-coordinate of \(P\) is \(y = 2a \sin^2 \theta\).

2. \(x = \frac{1}{t}, y = \sqrt{t} \ e^{-t} \Rightarrow \frac{dy}{dt} = t^{1/2} (-t^{-4} e^{-t}) + t^{-2/3} (\frac{1}{2} t^{-1/2} e^{-t}) = \frac{-2t + 1}{2t^{1/2} e^{-t}}, \frac{dx}{dt} = -\frac{1}{t^2}\), and 
\[\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2t + 1}{2t^{1/2} e^{-t}} \left(\frac{t^2}{1}\right) = \frac{2t^2}{2e^t} \left(\frac{t^2}{t^2 + 1}\right) = \frac{2t^2}{t^2 + 1}\].

4. \(x = t - t^{-1}, y = 1 + t^2; t = 1\). \(\frac{dy}{dt} = \frac{dx}{dt} = 2t, \frac{dx}{dt} = 1 + t^{-2} = \frac{t^2 + 1}{t^2}, \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t^2}{t^2 + 1}\).

When \(t = 1\), \((x, y) = (0, 2)\) and \(\frac{dy}{dx} = 1 = 1\), so an equation of the tangent to the curve at the point corresponding to \(t = 1\) is \(y = 2 = 1(x - 0), \text{ or } y = x + 2\).

6. \(x = \sin^2 \theta, y = \cos^3 \theta, \theta = \pi/6\). \(\frac{dy}{d\theta} = 3 \cos^2 \theta (-\sin \theta), \frac{dx}{d\theta} = 3 \sin^2 \theta \cos \theta, \text{ and}\)
\[\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-3 \cos^2 \theta \sin \theta}{3 \sin^2 \theta \cos \theta} = -\cot \theta. \text{ When } \theta = \pi/6, (x, y) = \left(\frac{1}{8}, \frac{3}{8}\sqrt{3}\right) \text{ and } \frac{dy}{dx} = -\cot(\pi/6) = -\sqrt{3}, \text{ so an equation of the tangent line to the curve at the point corresponding to } \theta = \pi/6 \text{ is } y - \frac{3}{8}\sqrt{3} = -\sqrt{3}(x - \frac{1}{8}), \text{ or } y = -\sqrt{3}x + \frac{1}{8}\sqrt{3}.

8. (a) \(x = 1 + \sqrt{t}, y = e^t; (2, e)\). \(\frac{dy}{dt} = e^t \cdot 2t, \frac{dx}{dt} = \frac{1}{2\sqrt{t}}, \text{ and } \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2te^t}{1/(2\sqrt{t})} = 4t^{3/2} e^t. \text{ At } (2, e),
\[x = 1 + \sqrt{t} = 2 \Rightarrow \sqrt{t} = 1 \Rightarrow t = 1 \text{ and } \frac{dy}{dx} = 4e, \text{ so an equation of the tangent is } y - e = 4e(x - 2), \text{ or } y = 4ex - 7e.

(b) \(x = 1 + \sqrt{t} \Rightarrow \sqrt{t} = x - 1 \Rightarrow t = (x - 1)^2, \text{ so } y = e^{x^2} = e^{(x-1)^2}, \text{ and } y' = e^{(x-1)^2} \cdot 4(x-1)^3.
\text{ At } (2, e), y' = e \cdot 4 = 4e, \text{ so an equation of the tangent is } y - e = 4e(x - 2), \text{ or } y = 4ex - 7e.

14. \(x = t^2 + 1, y = e^t - 1 \Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = e^t, \frac{d^2y}{dx^2} = \frac{d}{dt} \left(\frac{dy/dx}{dx/dt}\right) = \frac{2te^t - e^t \cdot 2}{(2t)^2} = \frac{2e^{t}(t - 1)}{(2t)^3} = \frac{e^t(t - 1)}{4t^3}.
\text{ The curve is } CU \text{ when } \frac{d^2y}{dx^2} > 0, \text{ that is, when } t < 0 \text{ or } t > 1.
16. $x = \cos 2t, \ y = \cos t, \ 0 < t < \pi$.

$$\frac{dy}{dx} = \frac{dy}{dt} \frac{dt}{dx} = \frac{-\sin t}{-2 \sin 2t} = \frac{\sin t}{2 \cdot 2 \sin t \cos t} = \frac{1}{4 \cos t} = \frac{1}{4} \sec t, \ so \ \frac{d^2y}{dx^2} = \frac{\frac{d}{dt} \left( \frac{dy}{dx} \right)}{\frac{dx}{dt}} = \frac{\frac{1}{4} \sec t \tan t}{-4 \sin t \cos t} = -\frac{1}{16} \sec^2 t.$$

The curve is CU when $\sec^2 t < 0 \iff \sec t < 0 \iff \cos t < 0 \iff \frac{\pi}{2} < t < \pi$.

28. $x = a \cos^2 \theta, \ y = a \sin^2 \theta$.

(a) $\frac{dx}{d\theta} = -3a \cos^2 \theta \sin \theta, \ \frac{dy}{d\theta} = 3a \sin^2 \theta \cos \theta, \ so \ \frac{dy}{dx} = -\frac{\sin \theta}{\cos \theta} = -\tan \theta$.

(b) The tangent is horizontal $\iff dy/dx = 0 \iff \tan \theta = 0 \iff \theta = n\pi \iff (x, y) = (\pm a, 0)$.

The tangent is vertical $\iff \cos \theta = 0 \iff \theta$ is an odd multiple of $\frac{\pi}{2} \iff (x, y) = (0, \pm a)$.

(c) $\frac{dy}{dx} = \pm 1 \iff \tan \theta = \pm 1 \iff \theta$ is an odd multiple of $\frac{\pi}{4} \iff (x, y) = \left( \pm \frac{\sqrt{2}}{2} a, \pm \frac{\sqrt{2}}{2} a \right)$

[All sign choices are valid.]

29. $x = 2t^2, \ y = 1 + 4t - t^2 \Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4 - 2t}{6t^2}$. Now solve $\frac{dy}{dx} = 1 \Rightarrow \frac{4 - 2t}{6t^2} = 1 \Rightarrow 6t^2 + 2t - 4 = 0 \iff 2(3t - 2)(t + 1) = 0 \iff t = \frac{2}{3} \ or \ t = -1$. If $t = \frac{2}{3}$, the point is $(\frac{16}{27}, \frac{20}{9})$, and if $t = -1$, the point is $(-2, -4)$.

33. The curve $x = 1 + e^t, \ y = t - t^2 = t(1 - t)$ intersects the $x$-axis when $y = 0$,

that is, when $t = 0$ and $t = 1$. The corresponding values of $x$ are 2 and $1 + e$.

The shaded area is given by

$$\int_{t=0}^{t=1} (y^2 - y)x \, dx = \int_{t=0}^{t=1} [y(t) - 0]x'(t) \, dt = \int_0^1 (t - t^2)e^t \, dt$$

$$= \int_0^1 te^t \, dt - \int_0^1 t^2e^t \, dt = \int_0^1 te^t \, dt - [t^2e^t]_0^1 + 2 \int_0^1 te^t \, dt \quad \text{[Formula 97 or parts]}
$$

$$= 3 \int_0^1 te^t \, dt - (e - 0) = 3 \left[ (t - 1)e^t \right]_0^1 - e \quad \text{[Formula 96 or parts]}
$$

$$= 3[0 - (1)] - e = 3 - e$$

34. By symmetry, $A = 4 \int_0^2 \int_0^\pi/2 a \sin^2 \theta (-3a \cos^2 \theta \sin \theta) \, d\theta = 12a^2 \int_0^\pi/2 \sin^2 \theta \cos^2 \theta \, d\theta$. Now

$$\int \sin^4 \theta \cos^2 \theta \, d\theta = \int \sin^2 \theta \left( \frac{1}{2} \sin^2 2\theta \right) \, d\theta = \frac{1}{8} \int (1 - \cos 2\theta) \sin^2 2\theta \, d\theta$$

$$= \frac{1}{8} \left[ \frac{1}{2} (1 - \cos 2\theta) - \sin^2 2\theta \cos 2\theta \right]_0^\pi/2 = \frac{1}{16} \theta - \frac{1}{4} \sin 4\theta - \frac{1}{48} \sin^3 2\theta + C$$

so $\int_0^\pi/2 \sin^4 \theta \cos^2 \theta \, d\theta = \left[ \frac{1}{16} \theta - \frac{1}{64} \sin 4\theta - \frac{1}{48} \sin^2 2\theta \right]_0^\pi/2 = \frac{\pi}{32}$. Thus, $A = 12a^2 \left( \frac{\pi}{32} \right) = \frac{3}{8} \pi a^2$. 

Page 5
69. (a) $\phi = \tan^{-1}\left(\frac{dy}{dx}\right) \Rightarrow \frac{d\phi}{dt} = \frac{d}{dt}\tan^{-1}\left(\frac{dy}{dx}\right) = \frac{1}{1 + (dy/dx)^2}\left(\frac{d}{dt}\left(\frac{dy}{dx}\right)\right)$. But $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\dot{y}}{\dot{x}} \Rightarrow$

\[
\frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{\ddot{x}y - \ddot{y}x}{\dot{x}^2 + \dot{y}^2} \Rightarrow \frac{d\phi}{dt} = \frac{1}{1 + (\ddot{x}/\ddot{y})^2} \left(\frac{\ddot{x}y - \ddot{y}x}{\dot{x}^2 + \dot{y}^2}\right) = \frac{\ddot{x}\dot{y} - \ddot{y}\dot{x}}{\dot{x}^2 + \dot{y}^2}. 
\]

Using the Chain Rule, and the fact that $s = \int_0^t \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt \Rightarrow \frac{ds}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = (\dot{x}^2 + \dot{y}^2)^{1/2}$, we have that

\[
\frac{d\phi}{ds} = \frac{\dot{\phi}}{\dot{s}} = \frac{\frac{d\phi}{dt}}{\frac{ds}{dt}} = \frac{\ddot{x}\dot{y} - \ddot{y}\dot{x}}{(\dot{x}^2 + \dot{y}^2)^{3/2}} = \frac{\ddot{y} - \ddot{x}}{(\dot{x}^2 + \dot{y}^2)^{3/2}}.
\]

(b) $x = x$ and $y = f(x) \Rightarrow \dot{x} = 1$, $\ddot{x} = 0$ and $\dot{y} = \frac{dy}{dx}$, $\ddot{y} = \frac{d^2y}{dx^2}$.

So $\kappa = \frac{|1 \cdot (d^2y/dx^2) - 0 \cdot (dy/dx)|}{1 + (dy/dx)^2} = \frac{|d^2y/dx^2|}{1 + (dy/dx)^2}$.

71. $x = \theta - \sin \theta \Rightarrow \dot{x} = 1 - \cos \theta \Rightarrow \ddot{x} = \sin \theta$, and $y = 1 - \cos \theta \Rightarrow \dot{y} = \sin \theta \Rightarrow \ddot{y} = \cos \theta$. Therefore,

\[
\kappa = \frac{|\cos \theta - \cos^2 \theta - \sin^2 \theta|}{(1 - \cos \theta)^{3/2}} = \frac{|\cos \theta - (\cos^2 \theta + \sin^2 \theta)|}{(1 - 2\cos \theta + \cos^2 \theta + \sin^2 \theta)^{3/2}} = \frac{|\cos \theta - 1|}{(2 - 2\cos \theta)^{3/2}}.
\]

The top of the arch is characterized by a horizontal tangent, and from Example 2(b) in Section 10.2, the tangent is horizontal when $\theta = (2\pi - 1)\pi$, so take $\kappa = 1$ and substitute $\theta = \pi$ into the expression for $\kappa$: $\kappa = \frac{|\cos \pi - 1|}{(2 - 2\cos \pi)^{3/2}} = \frac{|-1 - 1|}{|2 - 2(-1)|^{3/2}} = \frac{1}{4}$.

72. (a) Every straight line has parametrizations of the form $x = a + vt$, $y = b + wt$, where $a$, $b$ are arbitrary and $v, w \neq 0$.

For example, a straight line passing through distinct points $(a, b)$ and $(c, d)$ can be described as the parametrized curve $x = a + (c - a)t$, $y = b + (d - b)t$. Starting with $x = a + vt$, $y = b + wt$, we compute $\dot{x} = v$, $\dot{y} = w$, $\ddot{x} = \ddot{y} = 0$, and $\kappa = \frac{|v \cdot 0 - w \cdot 0|}{(v^2 + w^2)^{3/2}} = 0$.

(b) Parametric equations for a circle of radius $r$ are $x = r \cos \theta$ and $y = r \sin \theta$. We can take the center to be the origin.

So $\dot{x} = -r \sin \theta \Rightarrow \ddot{x} = -r \cos \theta$ and $\dot{y} = r \cos \theta \Rightarrow \ddot{y} = -r \sin \theta$. Therefore,

\[
\kappa = \frac{|r^2 \sin^2 \theta + r^2 \cos^2 \theta|}{(r^2 \sin^2 \theta + r^2 \cos^2 \theta)^{3/2}} = \frac{r^2}{r^3} = \frac{1}{r}.
\]

And so for any $\theta$ (and thus any point), $\kappa = \frac{1}{r}$.

24. $4y^2 = x \Leftrightarrow 4(r \sin \theta)^2 = r \cos \theta \Leftrightarrow 4r^2 \sin^2 \theta - r \cos \theta = 0 \Leftrightarrow r(4r \sin^2 \theta - \cos \theta) = 0 \Leftrightarrow r = 0$ or $\frac{\cos \theta}{4 \sin^2 \theta} \Rightarrow r = 0$ or $r = \frac{1}{4} \cot \theta \csc \theta$. $r = 0$ is included in $r = \frac{1}{4} \cot \theta \csc \theta$ when $\theta = \frac{\pi}{2}$, so the curve is represented by the single equation $r = \frac{1}{4} \cot \theta \csc \theta$.

26. $xy = 4 \Leftrightarrow (r \cos \theta)(r \sin \theta) = 4 \Leftrightarrow r^2 \left(\frac{1}{4} \cdot 2 \sin \theta \cos \theta\right) = 4 \Leftrightarrow r^2 \sin 2\theta = 8 \Leftrightarrow r^2 = 8 \csc 2\theta$
32. $r = 1 + 2 \cos \theta$

34. $r = \ln \theta$, $\theta \geq 1$

36. $r = \cos 5\theta$

40. $r = 2 + \sin \theta$

44. $r^2 \theta = 1 \iff r = \pm 1/\sqrt{\theta}$ for $\theta > 0$
48.